Socio-cultural Evolution of Opinion Dynamics in Networked Societies

Subhadeep Chakraborty[1] and Matthew M. Mench[1,2]

[1] Department of Mechanical, Aerospace, and Biomedical Engineering, University of Tennessee, Knoxville, Tennessee
[2] Energy and Transportation Science Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, United States

Abstract. This paper introduces a modeling paradigm based on a language theoretic framework for stochastic simulation of decision-making in a social setting, where choices and decisions by individuals are increasingly being influenced by a person's online social interactions. In this paper, the dynamics of opinion formation in a networked society have been studied with a joint model that bridges micro-level decisions based on reward maximization and the corresponding social influences which alter the estimate of these reward values. The effect of long term government policies on the stability and dynamics of the population opinion and the effect of including an influencing agent group has been studied. Simulated results on a sample society demonstrate the major impact of a relatively small but sharply opinionated influencing group toward pushing the society toward a desired outcome.

Keywords: Socio Cultural Decision Modeling, Effective Influence, Behavioral Dynamics.

1 Introduction

The dynamics of opinion formation in a networked society has developed into an active field of study in the last few decades. In the context of econometrics, individual decision-making has been largely modeled as an optimal policy estimation problem, aimed towards maximization of some utility function defined over a discrete set of choices [1]. But at the same time, it has been argued [2] that the human decision mechanism is never entirely rational and independent of external (social, political, economic, fashion trends) influences.

This paper is an attempt to introduce a modeling paradigm based on a language-theoretic framework for stochastic simulation of a person's decision-making process in a social setting, where the said individuals are influenced by their social interactions, including online social networking, and global (political) events. It has been conjectured that individuals in a population, exhibiting such collective behavior as a result of making decisions largely influenced by the actions of others, lead to rapid unstable fluctuations in the society. These cultural, political and social *snowballing* effects are called *information cascades* [2].

S.J. Yang, A.M. Greenberg, and M. Endsley (Eds.): SBP 2012, LNCS 7227, pp. 78–86, 2012.
© Springer-Verlag Berlin Heidelberg 2012

The concept of *information cascades*, based on *observational learning theory* was formally introduced in a 1992 article by Bikhchandani S. et. al. [2]. Watts D.J. [3] has studied the origin of rare cascades in terms of a sparse, random network of interacting agents using *generating functions*. Statistical mechanics tools, such as the Ising model [4] as well as non-equilibrium statistical models [5] has been used extensively to model the spread of influence in a networked society. *Influence maximization*, deals with finding the optimal set of people in a society to start an information cascade. Approximate solutions to this problem have been studied by Kempe [6] using *submodular functions*. However, in all these models, individual decision logic is largely overlooked.

Econometric theory, in contrast, contains a vast array of analyses tools for discrete outcomes [1]. Here, emphasis is on the logic behind discrete choices, specifically on selecting an optimal action policy which will lead to the largest discounted average future reward, but, interactions and influences are neglected.

To the best of the authors' knowledge, there have been relatively few attempts to merge these two areas into a more realistic model of opinion evolution in networked societies. In this paper, both complex microscopic logic, as well as social interactions that affect and are affected by these micro-choices are jointly studied to determine the advent of macroscopic social patterns which are observable, metrizable and potentially controllable.

2 The Framework for Discrete Choice Modeling (DCM)

Assumption 1. ***Finite set of discrete choices***

At each instant, every individual in the network is faced with the same set of finite discrete choices – for example, to vote for political candidate 'A' or 'B' or not to vote at all. In Markov Decision Modeling [1] as well as in the current framework, the problem is posed as finding the optimal choice policy for maximizing the rewards gained as a result of one's own choices. However, in a network, the rewards are also functions of choices by every other individual in the community. This feedback dynamics add a further layer of intricacy.

Assumption 2. ***Equivalent normative (rational) perspective***

The term *social norm* refers to rules or expectations for behavior that are shared by members of a group or society. For the sake of simplicity it is assumed in the model that majority of the society subscribe to the same rational consensus about right or wrong as a basic fact of organized social life. By no means does this imply that each individual makes decisions or reacts to stimulus identically, but simply that a rational structure may be imposed on the behavior of individuals.

Assumption 3. ***Probabilistic individual decisions***

It is assumed that, even when presented with the exact same choices with the exact same pay-offs, different individuals, and possibly even the same individual, may probabilistically make alternate decisions; the only restriction being that the decision cannot be *deviant* from the *normative perspective* (Assumption 2).

Assumption 4. ***Two kinds (external and internal (ε)) of events***

It is assumed that the set of events may be classified as *external/global* and *internal/local*. External events affect everyone simultaneously, according to the logic dictated by normative perspective. However, an individual's personal choice influences his own decisions, and possibly his networked neighbors to an extent. Some external events can lead to uncontrollable transitions, as explained next.

2.1 Normative Perspective Modeled as a Probabilistic Finite State Automata (PFSA)

The assumption of *normative perspective* allows rational behavior, to be encoded as a PFSA. In this example, each individual faces the *internal* decision of supporting the existing government, the opposing group, or remain in a state of indecision. Additionally, the individual can reach a state of political advantage, or disadvantage, but the uncontrollable transition to these two states can only occur through an *external* event, namely, the success or failure of the revolution.

Table 1. List of PFSA States and Events

States	Description	Events	Description
I	State of indecision	g	A popular act by the government
R	State of supporting the revolution	$\tilde{g}$	An unpopular government act
G	State of supporting the government	ε	An internal decision
A	State of political advantage	s	Success of the revolution
D	State of political disadvantage	f	Failure of the revolution

Figure 1 gives a schematic of the assumed normative perspective encoded as a PFSA. It may be noticed that transitions such as $g : G \rightarrow R$ or $g : I \rightarrow R$ are unauthorized, since it is assumed that a favorable act by the government should not make anyone decide to join the opposing group. Also, the same event can cause alternate transitions from the same state; the actual transition will depend probabilistically on the measure of attractiveness of the possible target states.

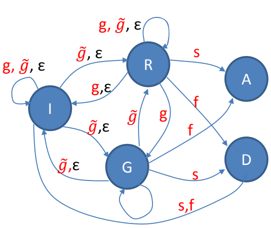


Fig. 1. Schematic of normative perspective coded as a PFSA

2.2 Rewards, Transition Costs and Probabilities

This section derives the logic behind probabilistic micro level choices by individuals. The probability of transitioning to a different state is dependent on the reachability of that state from the current state, the current event (external or internal), and also the relative degree of attractiveness of the target state. The state attractiveness measure is calculated using the concept of positive real measure attributed to a string of events [7], which is explained briefly here.

Let the discrete choice behavior be modeled as a PFSA as:

$$G_i \equiv (Q, \Sigma, \delta, q_i, Q_m) \tag{1}$$

where $Q = \{I, R, G, A, D\}$ is the finite set of choices with $|Q| = 5$ and the initial state $q_i \in Q = I$. The distribution of states may be represented as a coordinate vector of the form $\bar{v}_i$, defined as the $1 \times N$ vector $\left[v_1^i, v_2^i, ..., v_N^i\right]$, given by

$$v_j^i = \begin{cases} 1 \text{ if } i = j, \\ 0 \text{ if } i \neq j. \end{cases} \tag{2}$$

$\Sigma = \{\varepsilon, g, \tilde{g}, s, f\}$ is the (finite) alphabet of events with $|\Sigma| = 5$; the Kleene closure of Σ is denoted as Σ^*; the (possibly partial) function $\delta : Q \times \Sigma \times Q \to [0, 1]$ represents probabilities of state transitions and $\delta^* : Q \times \Sigma^* \times Q \to [0, 1]$ is an extension of δ; and $Q_m \subseteq Q$ is the set of marked (i.e., accepted) states.

Definition 1. *The reward from each state $\chi : Q \to [0, \infty)$ is defined as a characteristic function that assigns a positive real weight to each state q_i, such that*

$$\chi(q_j) \in \begin{cases} [0, \infty) & \text{if } q_j \in Q_m, \\ \{0\} & \text{if } q_j \notin Q_m. \end{cases} \tag{3}$$

Definition 2. *The event cost, conditioned on a PFSA state at which the event is generated, is defined as $\tilde{\pi} : \Sigma^* \times Q \to [0, 1]$ such that $\forall q_j \in Q, \forall \sigma_k \in \Sigma, \forall s \in \Sigma^*$,*

(1) $\tilde{\pi}[\sigma_k, q_j] \equiv \tilde{\pi}_{jk} \in [0, 1)$; $\sum_k \tilde{\pi}_{jk} < 1$;
(2) $\tilde{\pi}[\sigma, q_j] = 0$ *if* $\delta(q_j, \sigma, q_k) = 0 \, \forall k$; $\tilde{\pi}[\epsilon, q_j] = 1$;

The event cost matrix, ($\tilde{\Pi}$-matrix), is defined as: $\tilde{\mathbf{\Pi}} = \begin{bmatrix} \tilde{\pi}_{11} & \tilde{\pi}_{12} & \dots & \tilde{\pi}_{1m} \\ \tilde{\pi}_{21} & \tilde{\pi}_{22} & \dots & \tilde{\pi}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\pi}_{n1} & \tilde{\pi}_{n2} & \dots & \tilde{\pi}_{nm} \end{bmatrix}$

The characteristic vector $\bar{\chi}$ is chosen based on the individual state's impact. For example, if the states represent various job choices, the remuneration from these jobs can serve as the characteristic vector. The event cost is an intrinsic property of the nominal perspective. The event cost is conceptually similar to the state-based conditional probability of Markov Chains, except $\sum_k \tilde{\pi}_{jk} = 1$ is not allowed to be satisfied. The condition $\sum_k \tilde{\pi}_{jk} < 1$ provides a sufficient condition for the existence of the real signed measure, as discussed in [7].

Definition 3. *The state transition function of the PFSA is defined as a function $\pi : Q \times Q \to [0, 1)$ such that $\forall q_j$, and $q_k \in Q$,*

(1) $\pi(q_j, q_k) = \sum_{\sigma\in\Sigma:\, \delta(q_j,\sigma,q_k)\neq 0} \tilde{\pi}(\sigma, q_j) \equiv \pi_{jk}$
(2) and $\pi_{jk} = 0$ *if* $\{\sigma \in \Sigma : \delta(q_j, \sigma) = q_k\} = \emptyset$.

The state transition matrix, (Π-matrix), is defined as: $\mathbf{\Pi} = \begin{bmatrix} \pi_{11} & \pi_{12} & \cdots & \pi_{1n} \\ \pi_{21} & \pi_{22} & \cdots & \pi_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{n1} & \pi_{n2} & \cdots & \pi_{nn} \end{bmatrix}$

2.3 Measure of Attractiveness of the States

A real measure ν_θ^i for state i is defined as $\nu_\theta^i = \sum_{\tau=0}^{\infty} \theta\,(1-\theta)^\tau\, \bar{v}^i \Pi^\tau \bar{\chi}$ (4)

where $\theta \in (0, 1]$ is a user-specified parameter and $\bar{v}^i$ is defined in Eqn. 2.

Remark 1. Physical Significance of Real Measure

Assuming that the current state of the Markov process is i, i.e., the state probability vector is $\bar{v}_i$, at an instant τ time-steps in the future, the state probability vector is given by $\bar{v}^i \Pi^\tau$. Further, the expected value of the characteristic function is given by $\bar{v}^i \Pi^\tau \bar{\chi}$. The measure of state i, described by Eqn. 4, is the weighted expected value of χ over all time-steps in the future for the Markov process that begins in state i. The weights for each time-step $\theta\,(1-\theta)^\tau$, form a decreasing geometric series (sum equals 1). The measure in vector form yields

$$\bar{\nu}_\theta = \theta\,(\mathbb{I} - (1-\theta)\,\Pi)^{-1}\,\bar{\chi} \qquad \text{and} \qquad \bar{\nu}_{norm} = \frac{1}{\sum_k \nu_k}\bar{\nu} \qquad (5)$$

Remark 2. The effect of θ

The parameter θ controls the rate at which the weights decrease with increasing values of τ. Large values of θ forces the weights to decay rapidly, thereby placing more importance to states reachable in the near future from the current state. In fact, $\theta = 1$ implies that $\nu_\theta^i = \chi^i$. On the other hand, small values of θ captures the interaction with a large neighborhood of connected states. As $\theta \to 0$, the dependence on the initial state i is slowly lost (provided Π is irreducible).

The probabilistic transition decisions are dictated by $\bar{\nu}_{norm}$. Higher measure for a state implies that the discounted expected reward from that state is higher; consequently the incentive to transition to that state is proportionately higher.

3 Influence Model

One of the most interesting facets of a networked society is the strong interdependence between rewards and popularity of choices. For example, the reward from joining the revolutionary group ($\chi(R)$) may be small at the initial stages, but as more and more people join, the estimate of $\chi(R)$ as well as the probability of success of the revolution $P(s)$, increases. One of the advantages of the proposed modeling paradigm is that this multi-step reasoning is built into the language theoretic structure. Also, the framework lends itself well to all conventional linear and non-linear influence models such as the Friedkin-Johnsen [8] model or the bounded confidence model developed by Krause [9].

In this paper, it is assumed that the influence is entirely through the characteristic function χ of the states. This assumption is based on the physical insight that the anticipated reward from a state is the most well-discussed and well-broadcast quantity in a social network. In addition, influence of mass media can be accounted for, by assuming that an unbiased reporter reports the mass opinion in the form of a unified reward for the different choices averaged over the entire population. This may be mathematically expressed as

$$\bar{\chi}^i(t+1) = f_i\bar{\chi}^i(t) + g_i\frac{1}{|\mathcal{N}_i|}\sum_{j\in\mathcal{N}_i}\bar{\chi}^j(t) + h_i\frac{1}{N}\sum_{j=1}^{N}\bar{\chi}^j(t) \qquad (6)$$

where, N is the size of the network, $\mathcal{N}_i$ is the set of first order neighbors of node i, f_i is that fraction of the i^{th} individuals opinion about potential value of the available states that is based on his past beliefs, g_i is the fraction derived from the opinion of his acquaintances (network neighbors) and h_i=1-f_i-g_i is the fraction formed due to the influence of mass media such as newspapers, television, etc.

4 Progression of the Social Dynamics

The scale-free BA extended model network created with the Pajek [11] software has been used to model the connectivity structure, since many of the real-world networks are conjectured to be scale-free, including the World Wide Web, biological networks, and social networks [10]. The parameters are listed below.

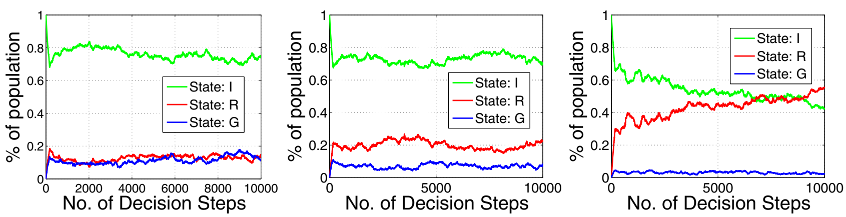
$N \rightarrow$ Number of vertices: 100
$m_0 \rightarrow$ Number of initial, disconnected nodes: 3
$m \rightarrow$ Number of edges to add/rewire at a time ($m \leq m_0$): 2
$p \rightarrow$ Probability to add new lines: 0.3333
$q \rightarrow$ Probability to rewire edges ($0 \leq q \leq 1-p$): 0.33335

Each individual is issued a random number drawn from a uniform distribution $U(0,1)$, representing the time remaining before that person makes a decision. This imposes an ordering on the list of people in the network. As soon as someone makes a decision, the time to his next decision, drawn from $U(0,1)$ is assigned, and the list is updated. Additionally, external events g and $\tilde{g}$ are also associated with a random time drawn from $U(a,b)$. Choosing a and b, the external events can be interspersed more, or less densely. At t_0, all individuals are initialized at state I. Initial values of the true reward vector $\bar{\chi}$ and the true event probabilities are fixed. Individuals receive a noisy estimate of the true probabilities and the rewards. At the time epoch t_k, when it is the i^{th} person's turn to make a decision, he updates his personal estimate of the reward vector according to the influence equation (Eqn. 6). He then calculates the degree of attractiveness of the states based on the normalized measure, using Eqn. 5. The transition probabilities are calculated as $P(q_{t_{k+1}} = q'|q_{t_k} = q, \sigma = \sigma') = \nu_{norm}(q')R(q, \sigma', q')$ where $R(q, \sigma', q') = 1$ if $\sigma' : q \rightarrow q'$ exists, otherwise 0. The only difference in the case of an external event such as $g, \tilde{g}, s$ or f is that everyone simultaneously updates their states rather than asynchronously, as in the case of internal events.

5 Simulation Scenarios and Results

5.1 Effect of Global Events

In this experiment, the probabilities of the good and bad external events, $P(g)$ and $P(\tilde{g})$ are varied to observe the effect of long term government policies on a population. Each simulation was run 30 times and the average of all the runs are showed in Fig. 2. When $P(g)$:$P(\tilde{g})$=1:1, the percentage of the population in states R and G are equal, and a large part of the population remains undecided (Fig. 2a). In the absence of any deadline for making a decision this result is only to be expected. As the government starts to push more and more unpopular policies, opinions bifurcate and the revolutionary group starts to become more popular. An interesting aspect of this result is that there is a threshold in the fraction of perceived negative acts by the government, above which the popularity of state R ramps up at a steady rate until everybody converges to a unipolar opinion (Fig. 2c). Below this threshold, order is maintained even when the government is not particularly popular (Fig. 2b).



(a) $P(g)$:$P(\tilde{g})$=1:1 (b) $P(g)$:$P(\tilde{g})$=3:7 (c) $P(g)$:$P(\tilde{g})$=1:9

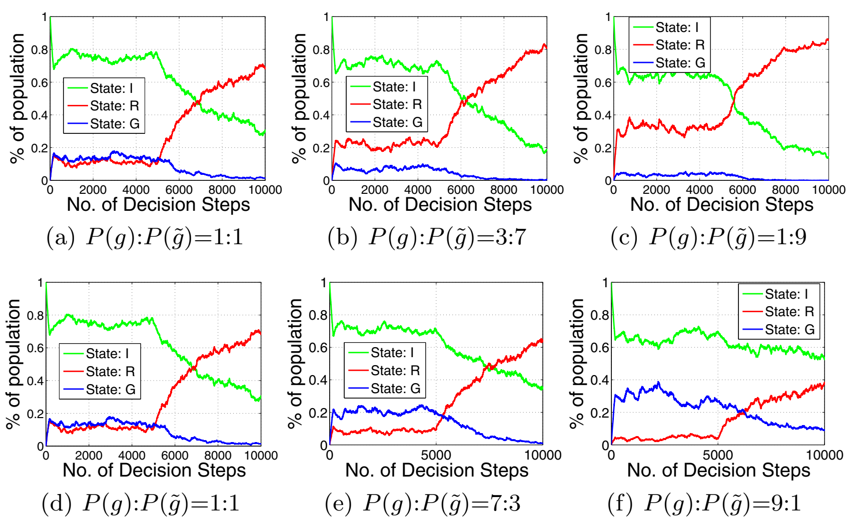
Fig. 2. Effect of external events on population opinion without influence group

5.2 Effect of an Influence Group

An interesting question in these social experiments is whether the mass opinion can be affected by adding a group of agents in the network with a premeditated agenda. These agents passively broadcast their opinion about the values of different decision states. In this paper, the influence has been studied through simulations in two different situations.

In the first case, starting from an even distribution $P(g)$:$P(\tilde{g})$=1:1, the proportion of unpopular acts are increased in two stages, $P(g)$:$P(\tilde{g})$=3:7 and 1:9

respectively. In the second case, the proportion of popular acts exceed unpopular acts and $P(g)$:$P(\tilde{g})$ is designed to be equal to 7:3 and 9:1. As a preliminary study, a 3 person group of external agents have been added to a total networked group of 100. This group randomly chooses people to form links, the probability of forming a link being 0.25. At decision step 5000, this influence group is activated, with all external agents initialized to state R. Themselves being in state R and broadcasting a high reward value for R, this group indirectly starts convincing its first order neighbors to increase their personal estimate of $\chi(R)$. Furthermore, in this particular setting, probability of success of the revolution $P(s)$ being linked to the percentage of the population in R, individuals develop a higher estimate of $P(s)$ as a result of being associated with these influence group members. Since the event probabilities affect the estimate of the state measure (μ_{norm}), more people probabilistically convert to state R. In fact, from the simulation a steady trickle of people are observed to convert to state R starting from step $t_k = 5000$. Gradually, the population converges to a state of revolution. As expected, when the government policies are unpopular, the opinion is already in favor of revolution, resulting in a faster transition (Fig. 3a, 3b and 3c) . But interestingly, even when the government decisions were more benevolent than harmful, inclusion of the influence group in the society can have destabilizing effects eventually leading to a social upheaval (Fig. 3d, 3e and 3f).



(a) $P(g)$:$P(\tilde{g})$=1:1 (b) $P(g)$:$P(\tilde{g})$=3:7 (c) $P(g)$:$P(\tilde{g})$=1:9

(d) $P(g)$:$P(\tilde{g})$=1:1 (e) $P(g)$:$P(\tilde{g})$=7:3 (f) $P(g)$:$P(\tilde{g})$=9:1

Fig. 3. Effect of external events on population opinion with $|I| = 3$

6 Conclusion and Future Work

This paper introduces a modeling paradigm based on a language-theoretic framework for stochastic simulation of a person's decision-making process in a social setting. It has been conjectured that, even though every individual in a network is influenced to by his/her social interactions, this is not blind following - rather, the influence is through altering the estimate of values or rewards to be obtained through different choices. Once these values are learned, individuals make their own optimal choices based on discounted reward maximization policies. In this paper, a specific problem which deals with the effect of long term government policies on the stability and dynamics of the population opinion has been studied. The effect of including an influencing agent group has also been observed. Specifically one case, where the influence group agenda matches the population sentiment, and another case, where it opposes, have been both reported. The results point to the existence of thresholds in the government policies below which it is possible to maintain an unpopular yet stable government, but above which, transition to an unstable state becomes inevitable. On one side of the threshold, the society ends up divided in opinion and reaches a multi-polar steady state. When the threshold is exceeded, the population converges to one unified opinion.

References

1. Greene, W.: Discrete Choice Modeling. In: Mills, T., Patterson, K. (eds.) The Handbook of Econometrics, Palgrave, London, vol. 2 (March 2009)
2. Bikhchandani, S., Hirshleifer, D., Welch, I.: A Theory of Fads, Fashion, Custom, and Cultural Change as Informational Cascades. The Journal of Political Economy 100(5), 992–1026 (1992)
3. Watts, D.J.: A simple model of global cascades on random networks. PNAS 99(9), 5766–5771 (2002), doi:10.1073/pnas.082090499
4. Dorogovtsev, S.N., Goltsev, A.V., Mendes, J.F.F.: Ising model on networks with an arbitrary distribution of connections. Physical Review E 66(1), 016104 (2002)
5. Klemm, K., Eguíluz, V.M., Toral, R., San Miguel, M.: Nonequilibrium transitions in complex networks: A model of social interaction. Physical Review E 67(2), 026120 (2003)
6. Kempe, D., Kleinberg, J., Tardos, É.: Maximizing the spread of influence through a social network. In: Proceedings of the Ninth ACM SIGKDD International Conference on Knowledge Discovery and Data Mining, pp. 137–146. ACM 1-58113-737-0
7. Ray, A.: Signed real measure of regular languages for discrete event supervisory control. International Journal of Control 78(12), 949–967 (2005)
8. Friedkin, N.E., Johnsen, E.C.: Social Influence Networks and Opinion Change. In: Advances in Group Processes, vol. 16, pp. 1–29. JAI Press Inc. (1999)
9. Hegselmann, R., Krause, U.: Opinion Dynamics and Bounded Confidence Models, Analysis and Simulation. Journal of Artificial Societies and Social Simulation 5 (2002), http://ideas.repec.org/a/jas/jasssj/2002-5-2.html
10. Barabási, A., Albert, R.: Emergence of scaling in random networks. Science 286(5439), 509–512 (1999)
11. Batagelj, V., Mrvar, A.: Pajek - Analysis and Visualization of Large Networks. In: Juenger, M., Mutzel, P. (eds.) Graph Drawing Software. Mathematics and Visualization, pp. 77–103. Springer, Heidelberg (2003) ISBN: 3-540-00881-0