Polymer-electrolyte fuel cells are a promising energy-conversion technology. Over the last several decades significant progress has been made in increasing their performance and durability, of which continuum-level modeling of the transport processes has played an important part. In this review, we examine the state-of-the-art modeling approaches, with a goal of elucidating the knowledge gaps and needs going forward in the field. In particular, the focus is on multiphase flow, especially in terms of understanding interactions at interfaces, and catalyst layers with a focus on the impacts of ionomer thin-films. Overall, we highlight where there is consensus in terms of modeling approaches as well as opportunities for further improvement and clarification, including identifying several critical areas for future research.

keywords: PEM fuel cell, mathematical modeling, transport phenomena, review, catalyst layer, multiphase flow, ionomer
<table>
<thead>
<tr>
<th>Section</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>4</td>
</tr>
<tr>
<td>Background</td>
<td>6</td>
</tr>
<tr>
<td>Model dimensionality</td>
<td>6</td>
</tr>
<tr>
<td>Empirical-based modeling</td>
<td>10</td>
</tr>
<tr>
<td>Electronic resistance</td>
<td>11</td>
</tr>
<tr>
<td>Protonic resistance</td>
<td>12</td>
</tr>
<tr>
<td>Thermal resistance</td>
<td>13</td>
</tr>
<tr>
<td>Diffusion resistance</td>
<td>13</td>
</tr>
<tr>
<td>Membrane transport</td>
<td>14</td>
</tr>
<tr>
<td>Along-the-channel solution</td>
<td>15</td>
</tr>
<tr>
<td>Basic governing equations</td>
<td>16</td>
</tr>
<tr>
<td>Thermodynamics</td>
<td>17</td>
</tr>
<tr>
<td>Material transport</td>
<td>18</td>
</tr>
<tr>
<td>Charge transport</td>
<td>22</td>
</tr>
<tr>
<td>Transport in the membrane</td>
<td>26</td>
</tr>
<tr>
<td>Momentum transport</td>
<td>27</td>
</tr>
<tr>
<td>Energy transport</td>
<td>28</td>
</tr>
<tr>
<td>Kinetics</td>
<td>31</td>
</tr>
<tr>
<td>Critical Issues in the Field</td>
<td>35</td>
</tr>
<tr>
<td>Multiphase flow</td>
<td>35</td>
</tr>
<tr>
<td>Incorporation of multiphase phenomena</td>
<td>37</td>
</tr>
<tr>
<td>Liquid/vapor/heat interactions</td>
<td>40</td>
</tr>
<tr>
<td>Microscale modeling and reconstruction</td>
<td>46</td>
</tr>
<tr>
<td>Pore-network modeling</td>
<td>50</td>
</tr>
<tr>
<td>Interfaces</td>
<td>54</td>
</tr>
<tr>
<td>Catalyst layer / microporous layer</td>
<td>55</td>
</tr>
<tr>
<td>Microporous layer / gas-diffusion layer</td>
<td>60</td>
</tr>
<tr>
<td>Gas-diffusion layer / flowfield</td>
<td>60</td>
</tr>
<tr>
<td>Ice formation</td>
<td>66</td>
</tr>
<tr>
<td>Catalyst-layer modeling</td>
<td>72</td>
</tr>
<tr>
<td>Microscale simulations and reconstruction</td>
<td>74</td>
</tr>
</tbody>
</table>
Introduction

Fuel cells may become the energy-delivery devices of the 21st century. Although there are many types of fuel cells, polymer-electrolyte fuel cells (PEFCs) are receiving the most attention for automotive and small stationary applications. In a PEFC, fuel and oxygen are combined electrochemically. If hydrogen is used as the fuel, it oxidizes at the anode releasing proton and electrons according to

$$H_2 \rightarrow 2H^+ + 2e^- \quad (1)$$

The generated protons are transported across the membrane and the electrons across the external circuit. At the cathode catalyst layer, protons and electrons recombine with oxygen to generate water

$$4H^+ + 4e^- + O_2 \rightarrow 2H_2O \quad (2)$$

Although the above electrode reactions are written in single step, multiple elementary reaction pathways are possible at each electrode. During the operation of a PEFC, many interrelated and complex phenomena occur. These processes include mass and heat transfer, electrochemical reactions, and ionic and electronic transport.

Over the last several decades significant progress has been made in increasing PEFC performance and durability. Such progress has been enabled by experiments and computation at multiple scales, with the bulk of the focus being on optimizing and discovering new materials for the membrane-electrode-assembly (MEA), composed of the proton-exchange membrane (PEM), catalyst layers, and diffusion-media (DM) backing layers. In particular, continuum modeling has been invaluable in providing understanding and insight into processes and phenomena that cannot be resolved or uncoupled through experiments. While modeling of the transport and related phenomena has progressed greatly, there are
still some critical areas that need attention. These areas include modeling the catalyst layer and multiphase phenomena in the PEFC porous media.

While there have been various reviews over the years of PEFC modeling\textsuperscript{1-6} and issues\textsuperscript{7-13} as well as numerous books and book chapters, there is a need to examine critically the field in terms of what has been done and what needs to be done. This review serves that purpose with a focus on transport modeling of PEFCs. As noted, this is not meant to be an exhaustive review of the very substantial literature on this topic, but to serve more as an examination and discussion of the state of the art and the needs going forward.

This review is organized as follows. First, some background introduction into PEFC transport modeling is accomplished including the general governing equations, modeling dimensionality, and a discussion on empirical modeling and the dominant mechanisms. Next, we critically examine multiphase-flow and catalyst-layer modeling. For the former, we will introduce several treatments and then focus on current issues including effective properties, some microscale modeling, phase-change behavior, and the impact and existence of interfaces. For catalyst-layer modeling, we discuss incorporating structural details into the modeling framework, and focus on consideration of ionomer thin-films, as well as transport in ionomer-free zones, and finally touch on the intersection between transport modeling and durability. The next section focuses on future perspectives including interactions between modeling and experiments, modeling variability, possible of open-source modeling, and an overall summary of the article.
Background

In this section, some background information is provided in order to orient the reader for the more detailed discussions below concerning multiphase-flow and catalyst-layer modeling and phenomena. As noted, the physics of most models is similar with the differences being in the scale of the model and phenomena investigated, treatment of the various transport properties, and boundary conditions. In this section, first a discussion of model dimensionality is made, followed by a general description and review of macroscale, empirical modeling. Finally, the general governing equations are presented.

Model dimensionality

Just as with scale, there is an issue of model dimensionality in that higher dimensional models better represent reality but at a great computational expense. Lower dimensionality model sacrifice some spatial fidelity but often allow for more complex physics to be incorporated. Due to increases in computational power, more multidimensional models are being employed, with perhaps the ideal tradeoff being the 1+2-D model framework.

Zero-dimensional (0-D) models relate system variables such as cell voltage, current, temperature, pressure, gas flow rate or any other property using simple empirical correlations without any consideration of spatial domain. 0-D models are used to determine kinetic and net ohmic resistance parameters from polarization performance curve.\textsuperscript{14-21} A typical 0-D model equation for polarization curve is

\[ V = U + b \log(i_o) - b \log(i) - R' i + b \log \left( 1 - \frac{i}{i_{lim}} \right) \] (3)
and accounts for the major losses as shown in Figure 1. The first and second term on the right correspond to the thermodynamic cell potential and exchange current density, $i_0$, respectively. The third term represents the loss in cell potential to kinetic resistance where $b$ is the Tafel slope. The fourth term accounts for the loss to ohmic resistance, $R'$, and the last represents the limiting current caused by concentration overpotential. As 0-D models do not provide fundamental understanding of PEFC operation, they have limited suitability for predicting performance for different operating conditions or optimizing the design. Empirically-based models are often 0-D models and can allow for some prediction of behavior of a specific material set under certain operating windows as discussed below.

![Figure 1 – Sample polarization curve showing dominant losses.](image-url)
Figure 2 – Schematic of spatial domains and associated modeling including the PEFC sandwich.

One dimensional (1-D) models describe the physical phenomena occurring in one spatial dimension typically across the membrane-electrode assembly. Comprehensive 1-D models incorporate electrochemical reaction at the porous electrodes, transport of gas and liquid species through porous gas-diffusion media (DM) or porous-transport layers (PTLs), and transport of charged species like electrons and protons. These models treat the cell as multiple layers bonded together. Proper interfacial internal boundary conditions are used to couple the different processes. Along-the-channel 1-D models focus on the transport and depletion of fuel and oxygen along the channel and its effects on current density.

Two-dimensional models use the 1-D model direction and the other direction can be classified into across-the-channel and along-the-channel models. Across-the-channel models focus on a cross section of the flow channel including the rib and channel. This approach addresses the effects of the solid rib and channel on the distribution of species such as electrons and water. As the rib is essentially a current collector, its contact with the gas-diffusion layer (GDL) blocks that portion of it to the gas channel. This reduces the mass transport to and from the electrode region directly underneath the rib. Along-the-
channel 2-D models incorporate the effect of fuel and oxygen depletion and water accumulation on the current distribution along the channel and across the cell sandwich. Distribution of water, temperature, and reacting species within the PEFC can be predicted by the model. This also helps in understanding the different types of channel configuration and flow direction which cannot be addressed with 1-D modeling. Most of the 2-D models assume that the serpentine flow channel, which is typically used in PEFCs, can be approximated as a straight channel. Such an assumption ignores the transport through the porous media under the ribs, but simplifies the model greatly. One can also use arguments of spatial separation to assume that the conditions in the cell sandwich only propagate and interact along the channel and not by internal gradients. This finding led to a group of models referred to as pseudo-2-D or 1+1-D models. Instead of solving the coupled conservation equations in a 2-D domain, the 1-D model is solved at each node along the channel. This reduces the computational requirement without the complexity of solving the equations in a 2-D domain.

The significant decrease in computing costs has promoted complex and computationally intensive numerical modeling including full 3-D models. In addition to understanding distribution of species and current, these models show the distribution of temperature and fluid in the 3-D spatial domain, especially the effect of cooling channels, channel cross section, and channel turn effects. These models typically use computational fluid dynamics (CFD). Similar to pseudo 2-D models, there is also a class of models termed pseudo 3-D or 1+2-D. In this formulation, the along-the-channel direction is only interacting at the boundaries between cell components, but instead of 1-D sandwich models, 2-D across-the-channel models are used. Such a structure is probably the best tradeoff between computational cost and model fidelity.

While the above classification is geared more towards macroscale modeling, similar delineations can be made on the microscale, where continuum equations are still used and remain valid. Typically, these
models are much smaller domains and are often of higher dimensionality since they are focused on specific phenomena. For example, modeling transport at the local scale of ions through a catalyst layer often requires 2-D and 3-D models to account for the correct geometry and geometry dependent physics. Finally, one can also examine multiscale models as being multidimensional. For example, as mentioned, there are 3-D models at the microscale that can determine properties which are subsequently used in a macroscale model that maybe is a 1-D model. Also, within the porous catalyst layer (see section below), typically one uses an expression or submodel for reaction into the reactive particle as well as across the domain, which, when taken together, can be considered as two separate dimensions.

**Empirical-based modeling**

As discussed above, there is extensive literature on modeling of transport processes in PEFCs and at various dimensions and physics. The approach currently taken by many fuel-cell developers is to first develop a comprehensive database from experiments conducted on a well-defined, representative material system. These experiments focus on *in-situ* and *ex-situ* measurement methods with resolution normal to the membrane to quantify transport processes at critical material interfaces, in addition to bulk-phase transport. Based on these component-level studies, models are developed in a simplified computational package that can be effectively used as an engineering tool, for assessment of the effects of material properties, design features, and operating conditions on PEFC performance. Such models are often classified as 0-D, although they can contain more detailed descriptions of the physics from the experiments. Thus, the model is of the form

\[ V = U - \eta_{\text{HOR}} - \eta_{\text{ORR}} - \eta_c - \eta_{\text{H}} - \eta_{\text{in}} \]  \hspace{1cm} (4)
where $\eta_{\text{HOR}}$ is the kinetic loss from the HOR, $\eta_{\text{ORR}}$ is the kinetic loss from the ORR, $\eta_{e^-}$ is the ohmic loss from electron transport, $\eta_{H^+}$ is the ohmic loss from proton transport, and $\eta_{\text{m-tran}}$ is the mass-transport loss. The reversible potential is often determined empirically from the open-circuit voltage or by a thermodynamic expression. The kinetic overpotentials can be determined based on experimental measurements of Tafel slopes (see equation 3). With regard to transport, researchers are focused on quantifying and modeling the last three transport terms in equation 4, which can be compared to the associated expressions given by equation 3. Electron, ion (proton), and mass transport are all strongly influenced by water transport and early work in collecting these losses for a comprehensive model was limited to operating conditions where the relative humidity was less than 100% or the impact of liquid-water accumulation was not accounted for explicitly.\textsuperscript{26} However, because of the effect of even small temperature gradients on water transport and phase change, the thermal transport resistance and the resulting saturation gradients are now being considered in parametric fuel-cell models.\textsuperscript{27,28} Below, we detail the various expressions and touch on empirical methods to obtain them.

**Electronic resistance**

The ohmic loss associated with the mobility of electrons is most strongly influenced by the contact resistances between the various diffusion layers with only a small contribution from the through-plane bulk electrical resistance. This resistance can be measured using the as-made GDM, typically consisting of both the GDL and microporous layer (MPL). To measure the resistance, two sheets of GDL with the MPLs facing each other are placed in a fixture with compression plates made of the flow field (current collector) material and geometry.\textsuperscript{29} For a given stress, this experiment results in a lumped electrical resistance that consists of all contact resistances along with the weighted average of the through-plane
and in-plane bulk resistances as they apply to the flowfield geometry. For a more detailed model or multi-dimensional approaches, the in-plane and contact resistances are measured independently.\textsuperscript{30} The bulk electrical resistance in the relatively thin electrode is negligible.

**Protonic resistance**

In addition to the electronic resistance through the solid phase, the ohmic resistance includes protonic resistance through the PEM. This resistance has a strong dependence on the RH in the adjacent electrodes and is only a weak function of temperature.\textsuperscript{31} The membrane conductance as it varies with RH can be characterized by \textit{in-situ}\textsuperscript{32} or \textit{ex-situ}\textsuperscript{33} methods. Furthermore, the sum of the electronic and protonic resistances is normally validated with AC impedance at high frequency during PEFC experiments at various operating conditions.

Resistance to proton transfer also occurs in the dispersed electrodes where a thin film of electrolyte is responsible for a lateral transport relative to the membrane plane. In addition to being a function of RH (as with the membrane), this resistance is also dependent on current density due to the variation in electrode utilization depth as current draw increases.\textsuperscript{34} The electrode effectiveness can be modeled theoretically with Tafel kinetics, and this is used to correct an area-based proton resistance in the anode and cathode electrodes which is commonly measured with H\textsubscript{2}/N\textsubscript{2} AC impedance\textsuperscript{31} and an associated transmission-line model.\textsuperscript{35} This method of characterizing the proton resistance in the electrode requires an assumption of uniform ionomer film thickness and connectivity. The ionomer film is difficult to characterize, and if this film was discontinuous with significant variations in thickness, the idealized proton resistance would under-predict the loss associated with proton mobility in the electrode. The ionomer film and its associated effects are discussed in more detail in a later section.
Thermal resistance

All of the voltage loss terms in equation 4 have some functionality with temperature. To predict accurately the reaction kinetics and the gas composition at the catalyst surface, the temperature profile through the PEFC sandwich must be established. There are several methods for characterizing the bulk and contact resistances of PEFC components as reviewed by Zamel and Li\textsuperscript{36} and Wang et al.\textsuperscript{6} The total thermal resistance is used to solve the heat equation in one of two dimensions based on the heat flux from the cathode catalyst layer. At high RH conditions, where proton resistance is minimized along with the product water flux from the cathode catalyst layer, such an analysis elucidates regions of phase change within the open volume of the diffusion layers and gas-delivery channels.

Diffusion resistance

There are several length scales for which diffusion resistance must be characterized. For a given total pressure in the flow channel, the first resistance to mass transport is encountered at the GDL interface. The convective mass-transfer coefficient, computed from the Sherwood number for a given flow rate and channel geometry is used for single phase conditions in the channel.\textsuperscript{37} The impact of liquid water in the gas channel is accomplish with a modified Sherwood number\textsuperscript{38} and by reducing the gas contact area using a surface-coverage ratio.\textsuperscript{39} In the GDM, the characterization of diffusion and governing parameters has been accomplished using various experiments combined with simple limiting-current or Fickian-type diffusion equations to get the values for use in equation 3 or 4.\textsuperscript{40-43} A more direct \textit{in-situ} measurement of oxygen diffusion resistance has been presented in previous work by Baker et al.\textsuperscript{37} and Caulk et al.\textsuperscript{44} that established a method and analysis using limiting-current measurements under dry and oversaturated conditions for use in equation 3. Although these measurements result in a
diffusion resistance while a two-phase condition exists in the PEFC assembly, the degree of saturation and its distribution is unknown; as a result, a direct correlation between saturation and the resulting change in the diffusion resistance is required to validate a simplified saturation model. The limiting-current analysis developed by Baker et al. can also be used to isolate the pressure-independent component of diffusion resistance (i.e., Knudsen diffusion and interfacial resistance through water/ionomer (combined with Henry’s law)). Studies have shown that this interfacial resistance is a significant component of voltage loss as it scales with reduced platinum loading and that it is much higher than expected based on oxygen permeability through bulk ionomer as discussed in more detail in another section below. Currently in a parametric model, this resistance is accounted for using an unrealistically thick ionomer layer with bulk (membrane) transport properties.

**Membrane transport**

Accurate prediction of the reactant partial pressure at the catalyst surface requires the solution of species throughout the PEFC assembly which is beyond the scope of empirical models. Although the thermal, electronic, protonic, and gas-phase transport resistances can be characterized, the water balance between the anode and cathode due to water permeation through the electrolyte must be accounted for. This water balance also complicates the prediction of phase change and most of the components of transport resistance are a function of the local liquid-water content. The key is to determine the net flux of water through the ionomer, which is a function of different driving forces including chemical potential and electro-osmosis driven by proton movement. For chemical-potential driven flow, there is still debate over the values of the transport coefficients and the possible existence of a humidity-dependent interfacial resistance in the membrane, where there is experimental support on both sides of the issue. The value of the electro-osmotic coefficient also varies significantly in
literature, and since it scales with current density it plays a critical role in predicting the water balance. Given the above difficulties, many parametric models use an empirical effective electro-osmotic coefficient.

**Along-the-channel solution**

The various resistances described above are assembled in a through-plane model with boundary conditions at the flow distributors that include total pressure, species concentration, temperature, and a reference potential of 0 V at the anode current collector. The cathode current collector potential is calculated based on equation 4, where electronic and protonic resistances are used to predict the ohmic losses and the diffusion and thermal resistances are used to predict the species partial pressures and temperature in the catalyst layers for calculation of the half-cell potentials, reaction kinetics, and membrane transport. This is typically solved in 1-D with average values over the geometric features.

Practical PEFC stacks have significant variation in the channel boundary conditions from the inlet to the outlet. To account for this, the parametric model is applied along-the-channel, assuming equipotential current collectors or by using a correlation for lateral potential difference through the current collector. This solution requires an accurate prediction of flow resistance in the channel. Although straight-forward for single-phase conditions, a two-phase pressure differential correlation is required once the flow distributor condition exceeds 100% RH. Additionally, PEFC systems typically operate with hydrogen stoichiometric ratios greater than 1.0, thereby requiring a methodology for recirculating exhaust anode flow. In this case the nitrogen and water content in the anode flow distributor due to membrane crossover must be accounted for as the diluted hydrogen feed stream will impact performance.
Basic governing equations

The above sections describe the methodologies and dimensionalities for modeling PEFCs based on a good amount of empiricism. To model PEFC behavior with more detailed physics at the continuum scale requires the use of overall conservation equations for mass, momentum, energy, and charge transport within the various subdomains or components. In addition, there is a need for expressions for overall kinetics and thermodynamics. The general physics equations are more or less known and used in their general forms, with the complications arising from the need to determine the correct boundary conditions, effective properties, and related transfer expressions. In this section, the general governing equations are presented including the conservation laws along with the general, well-known transport equations.

For a control segment, the general conservation equation for property $\psi$, representing any of the aforementioned transport processes, can be written as,

$$\frac{\partial \psi}{\partial t} + \nabla \cdot N_\psi = S_\psi$$

(5)

The first term represents the time dependent property and is neglected for description of steady-state operation, which is the favored approach to understand the mechanisms involved. However when it comes to application specific models – such as automobile applications involving start-stop cycles – dynamic models are better suited. While a vast majority of PEFC models are steady-state models, there are transient models that address specific transient phenomena such as, degradation mechanism,\textsuperscript{75,76} contamination effects,\textsuperscript{77} dynamic load-change effects,\textsuperscript{78,79} operational anomalies, start-stop cycles and freeze start\textsuperscript{5,80,81} and describe the transition of properties or system variables affecting PEFC operation.
The second term in equation 5 represents the change in the property \( \psi \) due to flux \((N)\) into or out of the control segment under study. The flux denotes the transfer of property driven by imbalances within the system and is the result of system adjusting itself to bring certain equilibrium.

The third term \((S)\) called the source term represents all the processes that cause generation or decay of the property driven by an imbalance within the control segment. For example, a supersaturated vapor phase may condense within the control segment and leads to a decrease in vapor-phase concentration and an increase in liquid-phase concentration. This term incorporates all other terms such as reaction terms and phase-change terms that are not captured by the flux. The term couples different conservation laws within the segment.

**Thermodynamics**

Although this is a review focused on transport modeling, one needs to be aware of the thermodynamics of the cell. From this perspective, a PEFC converts the intrinsic chemical energy of a fuel into electrical and heat energies. The associated Gibbs free energy results in a reversible cell voltage of 1.229 V at standard conditions,

\[
U^0 = \frac{\Delta G}{z_i F}
\]  

(6)

where \( z_i \) is the charge number of species \( i \) and \( F \) is Faraday’s constant. Similarly, one can define the enthalpy potential (1.48 V at standard conditions) as

\[
U_H = \frac{\Delta H}{z_i F}
\]

(7)

and thus the total heat released by the PEFC can be given by
where $V$ is the cell potential. Thus, if the cell potential equals the enthalpy potential, there is no net heat loss/gain.

These thermodynamic-based potentials can be amended to account for changes in temperature, for example,

$$U = U^0 + \Delta U = U^0 + \frac{\Delta S}{z_i F} (T - T^0)$$

(9)

Similarly, the impact of the departure of the reactants from standard conditions can also be captured by a Nernst equation,

$$U = U^0 - \frac{RT}{2F} \ln \left( \frac{p_{H_2} \sqrt{p_{O_2}}}{p_{H_2O}} \right)$$

(10)

where $p_i$ is the partial pressure of species $i$, and $R$ is the ideal-gas constant. The use of a reference potential using the Nernst equation versus a standard one corrected only for temperature can impact the overall surface overpotential and thus one needs to understand which is being used as discussed in the kinetics section. Also, from thermodynamics, the equilibrium and enthalpy potentials depend on the phase of water produced (i.e., liquid or vapor), and in this review it is assumed that water is produced in the condensed (liquid) phase next to the membrane. Since the associated liquid and vapor cell potentials are related by the vapor pressure of water, there is no issue in assuming a vapor product as long as the heat of vaporization is accounted for in the energy balance.

**Material transport**
The conservation of material can be written as in equation 5 except that the physical quantity \( \psi \) could be \( p \) – partial pressure of gas, \( c \) – concentration of solution, \( x \) – mole fraction of particular species, or \( \rho \) – density of fluid. However, for the case of a mixture in a multiphase system, it is necessary to write material balances for each of the component in each phase \( k \), which in summation can still govern the overall conservation of material,

\[
\frac{\partial \varepsilon_k c_{i,k}}{\partial t} = -\nabla \cdot \mathbf{N}_{i,k} - \sum_h a_{i,k} s_{i,k,h} \frac{i_{h,1-k}}{n_h F} + \sum_l s_{i,k,l} \sum_{p \neq k} a_{k,p} r_{i,k-l,p} + \sum_g s_{i,k,g} \varepsilon_k R_{g,k} \tag{11}
\]

In the above expression, \( \varepsilon_k \) is the volume fraction of phase \( k \), \( c_{i,k} \) is the concentration of species \( i \) in phase \( k \), and \( s_{i,k,l} \) is the stoichiometric coefficient of species \( i \) in phase \( k \) participating in heterogeneous reaction \( l \), \( a_{k,p} \) is the specific surface area (surface area per unit total volume) of the interface between phases \( k \) and \( p \), \( i_{h,1-k} \) is the normal interfacial current transferred per unit interfacial area across the interface between the electronically conducting phase and phase \( k \) due to electron-transfer reaction \( h \), and is positive in the anodic direction.

The term on the left side of the equation is the accumulation term, which accounts for the change in the total amount of species \( i \) held in phase \( k \) within a differential control volume over time. The first term on the right side of the equation keeps track of the material that enters or leaves the control volume by mass transport as discussed in later sections. The remaining three terms account for material that is gained or lost (\( i.e., \) source terms, \( S_{yi} \) in equation 5). The first summation includes all electron-transfer reactions that occur at the interface between phase \( k \) and the electronically conducting phase 1; the second summation accounts for all other interfacial processes that do not include electron transfer like evaporation or condensation; and the final term accounts for homogeneous chemical reactions in phase \( k \). It should be noted that in terms of an equation count, for \( n \) species there are only
$n-1$ conservation equations needed since one can be replaced by the sum of the other ones, or, similarly, by the fact that the sum of the mole fractions equals 1,

$$\sum_i x_i = 1 \quad (12)$$

In the above material balance (equation 11), one needs an expression for the flux or transport of material. Often, this expression stems from considering only the interactions of the various species with the solvent

$$N_{i,k} = -D_i \nabla c_{i,k} + c_i v_k \quad (13)$$

where $v_k$ is the mass-averaged velocity of phase $k$

$$v_k = \frac{\sum_i M_i N_{i,k}}{\rho_k} \quad (14)$$

One can see that if convection is neglected, equation 13 results in Fick’s law. Substitution of equation 13 into equation 11 results in the equation for convective diffusion, which, in terms of mass, is

$$\frac{\partial (\epsilon_k \rho_k w_i)}{\partial t} + \nabla \cdot (v_k \rho_k w_i) = \nabla \cdot (\rho_k D_i^{\text{eff}} \nabla w_i) + S_i \quad (15)$$

which is often used in CFD simulations. In the above expression, the reaction source terms are not shown explicitly, $w_i$ is the mass fraction of species $i$, and the superscript ‘eff’ is used to denote an effective diffusion coefficient due to different phenomena or phases as discussed later in this review. For example, to account for Knudsen diffusion primarily in the MPLs and catalyst layers, one can use a diffusion coefficient that is a parallel resistance$^{82}$

$$D_i^{\text{eff}} = \frac{1}{\frac{1}{D_i} + \frac{1}{D_{k_i}}} \quad (16)$$
where the Knudsen diffusivity is given by

\[ D_K = \frac{d}{3} \left( \frac{8RT}{\pi M_i} \right)^{1/2} \]  

(17)

where \( d \) is the pore diameter. This diffusion coefficient is independent of pressure whereas ordinary diffusion coefficients have an inverse dependence on pressure.

If the interactions among the various species are important, then equation 13 needs to be replaced with the multicomponent Stefan-Maxwell equations that account for binary interactions among the various species

\[ \nabla x_{i,k} = -\frac{x_{i,k}}{RT} \left( \frac{M_i}{\rho_k} - \bar{V}_i \right) \nabla P_k + \sum_{j=i}^n \frac{x_{i,k} N_{j,k} - x_{j,k} N_{i,k}}{e_k c_{T,k} D_{i,j}} - \frac{N_{i,k}}{e_k c_{T,k} D_{K,i}} \]  

(18)

where \( x_i \) and \( M_i \) are the mole fraction and molar mass of species \( i \), respectively, \( D_{i,j}^{\text{eff}} \) is the effective binary diffusion coefficient between species \( i \) and \( j \), and \( c_{T,k} \) is the total concentration of species in phase \( k \) as derived from the ideal-gas law. The first term on the right side accounts for pressure diffusion (e.g., in centrifugation) which often can be ignored, but, on the anode side, the differences between the molar masses of hydrogen and water means that it can become important in certain circumstances.\(^{83}\) The second term on the right side stems from the binary collisions between various components. For a multicomponent system, equation 18 results in the correct number of transport properties that must be specified to characterize the system, \( \frac{1}{2} n(n-1) \), where \( n \) is the number of components and the \( \frac{1}{2} \) is because \( D_{i,j}^{\text{eff}} = D_{j,i}^{\text{eff}} \) by the Onsager reciprocal relationships. Finally, the third term on the right side represented Knudsen diffusion effects when incorporated rigorously and using the solid as the reference velocity.\(^{84}\)
The form of equation 18 is essentially an inverted form of the type of equation 13, since one is not writing the flux in terms of a material gradient but the material gradient in terms of the flux. This is not a problem if one is solving the equations as written; however, many numerical packages require a second-order differential equation (e.g., see equation 15). To do this with the Stefan-Maxwell equations, inversion of them is required. For a two-component system where the pressure-diffusion is negligible, one arrives at equation 13. For higher numbers of components, the inversion becomes cumbersome and analytic expressions are harder to obtain, resulting oftentimes in numerical inversion. In addition, the inversion results in diffusion coefficients which are more composition dependent. For example, Bird et al. show the form for a three-component system.85

**Charge transport**

The conservation equation for charged species is an extension of the conservation of mass. Taking equation 11 and multiplying by $z_iF$ and summing over all species and phases while noting that all reactions are charge balanced yields

$$\frac{\partial}{\partial t} F \sum_k \sum_i z_i c_{i,k} = -\nabla \cdot F \sum_k \sum_i z_i N_{i,k}$$

(19)

where the charge and current densities can be defined by

$$\rho_e = F \sum_k \sum_i z_i c_{i,k}$$

(20)

and

$$i_k = F \sum_i z_i N_{i,k}$$

(21)
respectively. Because a large electrical force is required to separate charge over an appreciable distance, a volume element in the electrode will, to a good approximation, be electrically neutral; thus one can assume electroneutrality for each phase

$$\sum z_i c_{i,k} = 0 \quad (22)$$

The assumption of electroneutrality implies that the diffuse double layer, where there is significant charge separation, is small compared to the volume of the domain, which is normally (but not necessarily always as discussed later) the case. The general charge balance (equation 19), assuming electroneutrality and the current definition (equation 21) becomes

$$\sum \nabla \cdot \mathbf{i}_k = 0 \quad (23)$$

While this relationship applies for almost all of the modeling, there are cases where electroneutrality does not strictly hold, including for some transients and impedance measurements, where there is charging and discharging of the double layer, as well as simulations at length scales within the double layer (typically on the order of nanometers) such as reaction models near the electrode surface. For these cases, the correct governing charge conservation results in Poisson’s equation,

$$\nabla^2 \Phi = \frac{\rho_e}{\varepsilon_0} \quad (24)$$

where $\varepsilon_0$ is the permittivity of the medium. For the diffuse part of the double layer, often a Boltzmann distribution is used for the concentration of species $i$

$$c_i = c_{i,\infty} \exp\left(-\frac{z_i F \Phi}{RT}\right) \quad (25)$$
where \( \Phi \) is the potential as referenced to the bulk solution (i.e., having concentration \( c_{i,\infty} \)). To charge this double layer, one can derive various expressions for the double-layer capacitance depending on the adsorption type, ionic charges, etc.,\(^8\) where the differential double-layer capacitance is defined as

\[
C_d = \left( \frac{\partial q}{\partial \Phi} \right)_{\mu_i, T}
\]

where \( q \) is the charge in the double layer and the differential is at constant composition and temperature. To charge the double layer, one can write an equation of the form

\[
i = C_d \frac{\partial \Phi}{\partial t}
\]

where the charging current will decay with time as the double layer becomes charged.

For the associated transport of charge, models can use either a dilute-solution or concentrated-solution approach. In general, the concentrated-solution approach is more rigorous but requires more knowledge of all of the various interactions (similar to the material-transport-equation discussion above). For the dilute-solution approach, one can use the Nernst-Planck equation,

\[
N_{i,k} = -z_i u_i F c_{i,k} \nabla \Phi_k - D_i \nabla c_{i,k} + c_{i,k} v_k
\]

where \( u_i \) is the mobility of species \( i \). In the equation, the terms on the right side correspond to migration, diffusion, and convection, respectively. Multiplying equation 28 by \( z_i F \) and summing over the species \( i \) in phase \( k \),

\[
F \sum_i z_i N_{i,k} = -F^2 \sum_i z_i^2 u_i c_{i,k} \nabla \Phi_k - F \sum_i z_i D_i \nabla c_{i,k} + F \sum_i z_i c_{i,k} v_k
\]

and noting that the last term is zero due to electroneutrality (convection of a neutral solution cannot move charge) and using the definition of current density (equation 21), one gets
\[ i_k = -\kappa_k \nabla \Phi_k - F \sum_i z_i D_i \nabla c_{i,k} \]  

(30)

where \( \kappa_k \) is the conductivity of the solution of phase \( k \)

\[ \kappa_k = F^2 \sum_i z_i^2 c_{i,k} u_i \]  

(31)

When there are no concentration variations in the solution, equation 30 reduces to Ohm’s law,

\[ i_k = -\kappa_k \nabla \Phi_k \]  

(32)

This dilute-solution approach does not account for interaction between the solute molecules. Also, this approach will either use too many or too few transport coefficients depending on if the Nernst-Einstein relationship is used to relate mobility and diffusivity,

\[ D_j = RT \mu_i \]  

(33)

which only rigorously applies at infinite dilution. Thus, concentrated-solution theory is preferred unless there are too many unknown parameters and transport properties.

The concentrated-solution approach for charge utilizes the same underpinnings as that of the Stefan-Maxwell equation, which starts with the original equation of multicomponent transport

\[ d_i = c_i \nabla \mu_i = \sum_{j \neq i} K_{i,j} (v_j - v_i) \]  

(34)

where \( d_i \) is the driving force per unit volume acting on species \( i \) and can be replaced by a electrochemical-potential gradient of species \( i \), and \( K_{i,j} \) are the frictional interaction parameters between species \( i \) and \( j \). The above equation can be analyzed in terms of finding expressions for \( K_{i,j} \)’s, introducing the concentration scale including reference velocities and potential definition, or by inverting the equations and correlating the inverse friction factors to experimentally determined properties. If one uses a diffusion coefficient to replace the drag coefficients,
where $$D_{ij}$$ is an interaction parameter between species $$i$$ and $$j$$ based on a thermodynamic driving force, then the multicomponent equations look very similar to the Stefan-Maxwell ones (equation 18). In addition, using the above definition for $$K_{ij}$$ and assuming that species $$i$$ is a minor component and that the total concentration, $$c_T$$, can be replaced by the solvent concentration (species 0), then equation 34 for species $$i$$ in phase $$k$$ becomes

$$N_{i,k} = -\frac{D_{ik}}{RT} c_{i,k} \nabla \mu_{i,k} + c_{i,k} \mathbf{v}_0$$  

(36)

This equation is very similar to the Nernst-Planck equation (28), except that the driving force is the thermodynamic electrochemical potential, which contains both the migration and diffusive terms.

**Transport in the membrane**

For the special case of the PEM (subscript 2), concentrated-solution theory can be used to obtain

$$\mathbf{i}_2 = -\kappa \nabla \Phi_2 - \frac{\kappa \xi}{F} \nabla \mu_w$$  

(37)

and

$$N_w = -\frac{\kappa \xi}{F} \nabla \Phi_2 - \left( \alpha + \frac{\kappa \xi^2}{F^2} \right) \nabla \mu_w$$  

(38)

where $$\xi$$ is the electro-osmotic coefficient and is defined as the ratio of flux of water to the flux of protons in the absence of concentration gradients, $$\alpha$$ is the transport parameter that can be related to a hydraulic pressure or concentration gradient through the chemical-potential driving force,

$$\nabla \mu_w = RT \nabla \ln a_w + \bar{v}_w \nabla p$$  

(39)
where $a_w$, $V_w$, and $p$ are the activity, molar volume, and hydraulic pressure of water, respectively. As mentioned, there is still a lot of debate over the functional forms of the transport properties of the membrane, as well as the existence of a possible interfacial resistance.\textsuperscript{50-67,89,90} To account for such an interface, the membrane water-uptake boundary condition is altered to include a mass-transfer coefficient instead of assuming an equilibrium isotherm directly

$$N = k_{mt}(a_{in} - a_{out})$$  \hspace{1cm} (40)

where in and out refer to the water activities directly inside and outside of the membrane interface and $k_{mt}$ is a mass-transfer coefficient. This approach is the same as including a surface reaction (e.g., condensation) at the membrane interface.

**Momentum transport**

Due to the highly coupled nature of momentum conservation and transport, both are discussed below. Also, the momentum or volume conservation equation is highly coupled to the mass or continuity conservation equation (equation 11). Newton’s second law governs the conservation of momentum and can be written in terms of the Navier-Stokes equation\textsuperscript{82}

$$\frac{\partial(\rho_k \mathbf{v}_k)}{\partial t} + \nabla \cdot (\rho_k \mathbf{v}_k) = -\nabla p_k + \mu_k \nabla^2 \mathbf{v}_k + S_m$$  \hspace{1cm} (41)

where $\mu_k$ and $\mathbf{v}_k$ are the viscosity and mass-averaged velocity of phase $k$ (equation 14), respectively. The transient term in the momentum conservation equation represents the accumulation of momentum with time and the second term describes convection of the momentum flux (which is often small for PEFC designs). The first two terms on the right side represent the divergence of the stress tensor and the last term represents other sources of momentum, typically other body forces like gravity or magnetic forces. However for PEFCs, these forces are often ignored and unimportant, \textit{i.e.} $S_m = 0$. 

27
It should be noted that for porous materials and multiphase flow, as discussed below, the Navier-Stokes equations are not used and instead one uses the more empirical Darcy’s law for the transport equation, \(^91,92\)

\[ \mathbf{v}_k = -\frac{k_k}{\mu_k} \nabla p_k \tag{42} \]

This transport equation can be used as a first-order equation or combined with a material balance (equation 11) to yield

\[ \frac{\partial (\rho_k \varepsilon_k)}{\partial t} = \nabla \cdot \left( -\rho_k \frac{k_k}{\mu_k} \nabla p_k \right) + S_m \tag{43} \]

which is similar to including it as a dominant source term. In the above expression, \(k_k\) is effective permeability of phase \(k\).

**Energy transport**

Throughout all layers of the PEFC, the same transport and conservation equations exist for energy with the same general transport properties, and only the source terms vary. The conservation of thermal energy can be written as

\[ \rho_k \hat{C}_{p_k} \left( \frac{\partial T_k}{\partial t} + \mathbf{v}_k \cdot \nabla T_k \right) + \left( \frac{\partial \ln \rho_k}{\partial \ln T_k} \right)_{\mu_k, v_k, k} \left( \frac{\partial p_k}{\partial t} + \mathbf{v}_k \cdot \nabla p_k \right) \]

\[ = Q_{k,p} - \nabla \cdot \mathbf{q}_k - \mathbf{r} : \nabla \mathbf{v}_k + \sum_i \mathbf{H}_{i,k} \nabla \cdot \mathbf{J}_{i,k} - \sum_i \mathbf{H}_{i,k} \mathbf{g}_{i,k} \tag{44} \]

In the above expression, the first term represents the accumulation and convective transport of enthalpy, where \(\hat{C}_{p_k}\) is the heat capacity of phase \(k\) which is a combination of the various components of that phase. The second term is energy due to reversible work. For condensed phases this term is
negligible, and an order-of-magnitude analysis for ideal gases with the expected pressure drop in a PEFC demonstrates that this term is negligible compared to the others. The first two terms on the right side of equation 44 represent the net heat input by conduction and interphase transfer. The first is due to heat transfer between two phases

\[ Q_{k,p} = h_{k,p} a_{k,p} (T_p - T_k) \]  (45)

where \( h_{k,p} \) is the heat-transfer coefficient between phases \( k \) and \( p \) per interfacial area. Most often this term is used as a boundary condition since it occurs only at the edges. However, in some modeling domains it may need to be incorporated as above. The second term is due to the heat flux in phase \( k \)

\[ q_k = -\sum_i \overline{H}_{i,k} J_{i,k} - k^\text{eff} T_k \]  (46)

where \( \overline{H}_{i,k} \) is the partial molar enthalpy of species \( i \) in phase \( k \), \( J_{i,k} \) is the flux density of species \( i \) relative to the mass-average velocity of phase \( k \)

\[ J_{i,k} = N_{i,k} - c_{i,k} v_k \]  (47)

and \( k^\text{eff} \) is the effective thermal conductivity of phase \( k \). The third term on the right side of equation 44 represents viscous dissipation, the heat generated by viscous forces, where \( \tau \) is the stress tensor. This term is also small, and for most cases can be neglected. The fourth term on the right side comes from enthalpy changes due to diffusion. Finally, the last term represents the change in enthalpy due to reaction

\[ \sum_i \overline{H}_{i,k} R_{i,k} = -\sum_h a_{1,k} \left[ \eta_{h_1,k} + \eta_{h_2,k} \right] - \sum_{p;k} \Delta H_p a_{k,p} r_{i,k-p} - \sum_g \Delta H_g R_{g,k} \]  (48)
where the expressions can be compared to those in the material conservation equation (11). The above reaction terms include homogeneous reactions, interfacial reactions (e.g., evaporation), and interfacial electron-transfer reactions. The irreversible heat generation is represented by the activation overpotential and the reversible heat generation is represented by the Peltier coefficient, \( \Pi_h \). The Peltier coefficient for charge-transfer reaction \( h \),

\[
\Pi_h \approx \frac{T}{n_h F} \sum_i S_{i,k,h} \overline{S}_{i,k} = \frac{T \Delta S_h}{n_h F}
\]

where \( \Delta S_h \) is the entropy of reaction \( h \). The above equation neglects transported entropy (hence the approximate sign), and the summation includes all species that participate in the reaction (e.g., electrons, protons, oxygen, hydrogen, water). While the entropy of the half-reactions that occur at the catalyst layers is not truly obtainable since it involves knowledge of the activity of an uncoupled proton, the Peltier coefficients have been measured experimentally for these reactions, with most of the reversible heat due to the 4-electron oxygen reduction reaction.\textsuperscript{93,94}

It is often the case that because of the intimate contact between the gas, liquid, and solid phases within the small pores of the various PEFC layers, that equilibrium can be assumed such that all of the phases have the same temperature for a given point in the PEFC (this assumption is discussed more in a later section). Assuming local equilibrium eliminates the phase dependences in the above equations and allows for a single thermal energy equation to be written. Neglecting those phenomena that are minor as mentioned above and summing over the phases, results in

\[
\sum_k \rho_k \dot{C}_{p,k} \frac{\partial T}{\partial t} = -\sum_k \rho_k \dot{C}_{p,k} \mathbf{v}_k \cdot \nabla T + \nabla \cdot \left( k_f \nabla T \right) + \sum_k i_k \mathbf{v}_k + \sum_k k_{\text{eff}} i_k + \sum_h i_h \left( \eta_h + \Pi_h \right) - \sum_h \Delta H_h r_h
\]

\( (50) \)
where the expression for Joule or ohmic heating has been substituted in for the third-term in the right side of equation 44

$$\sum_i \mathbf{J}_{i,k} \cdot \nabla H_{i,k} = -\mathbf{i}_k \cdot \nabla \Phi_k = \frac{\mathbf{i}_k \cdot \mathbf{i}_k}{\kappa_{\text{eff}}},$$  \hspace{1cm} (51)$$

In equation 50, the first term on the right side is energy transport due to convection, the second is energy transport due to condition, the third is the ohmic heating, the fourth is the reaction heats, and the last represents reactions in the bulk which include such things as vaporization/condensation and freezing/melting. Heat lost to the surroundings is only accounted for at the boundaries of the cell. In terms of magnitude, the major heat generation sources are the oxygen reduction reaction and the water phase changes, and the main mode of heat transport is through conduction.

**Kinetics**

Electrochemical kinetic expressions provide the transfer current in the general material balance (i.e., $i_{h,1-k}$ in equation 11). A typical electrochemical reaction can be expressed as

$$\sum_k \sum_i s_{i,k,h} M_i^{z_i} \rightarrow n_h e^-$$  \hspace{1cm} (52)$$

where $s_{i,k,h}$ is the stoichiometric coefficient of species $i$ residing in phase $k$ and participating in electron-transfer reaction $h$, $n_h$ is the number of electrons transferred in reaction $h$, and $M_i^{z_i}$ represents the chemical formula of $i$ having valence $z_i$. According to Faraday’s law, the flux or species $i$ in phase $k$ and rate of reaction $h$ is related to the current as

$$N_{i,k} = \sum_h r_{h,i,k} = \sum_h s_{i,k,h} \frac{i_h}{n_h F},$$  \hspace{1cm} (53)$$
For modeling purposes, especially with the multi-electron transport of species, it is often easiest to use a semi-empirical equation to describe the reaction rate, namely, the Butler-Volmer equation,\textsuperscript{86,95}

\begin{equation}
    i_h = i_0 \exp \left( \frac{\alpha_a F}{RT} (\Phi_k - \Phi_p - U_h^\text{ref}) \right) \prod_i \left( \frac{a_i}{a_i^\text{ref}} \right)^{\gamma_{i,k,p}} - \exp \left( \frac{-\alpha_c F}{RT} (\Phi_k - \Phi_p - U_h^\text{ref}) \right) \prod_i \left( \frac{a_i}{a_i^\text{ref}} \right)^{-\gamma_{i,k,p}} \right]^{(54)}
\end{equation}

where \(i_h\) is the transfer current between phases \(k\) and \(p\) due to electron-transfer reaction \(h\), the products are over the anodic and cathodic reaction species, respectively, \(\alpha_a\) and \(\alpha_c\) are the anodic and cathodic transfer coefficients, respectively, and \(i_0\) and \(U_h^\text{ref}\) are the exchange current density per unit catalyst area and the potential of reaction \(h\) evaluated at the reference conditions and the operating temperature, respectively.

In the above expression, the composition-dependent part of the exchange current density is explicitly written, with the multiplication over those species participating in the anodic or cathodic direction. The reference potential is determined by thermodynamics as described above (e.g., equation 10). The term in parentheses in equation 27 can be written in terms of an electrode overpotential

\begin{equation}
    \eta_h = \Phi_k - \Phi_p - U_h^\text{ref} \tag{55}
\end{equation}

If the reference electrode is exposed to the conditions at the reaction site, then a surface or kinetic overpotential can be defined

\begin{equation}
    \eta_{s_h} = \Phi_k - \Phi_p - U_h \tag{56}
\end{equation}

The surface overpotential is the overpotential that directly influences the reaction rate across the interface. Comparing equations 29 and 30, one can see that the electrode overpotential contains both a concentration and a surface overpotential for the reaction.

For the HOR occurring at the anode, equation 54 can be written as
\[ i_{\text{HOR}} = i_{\text{HOR}}^0 \left[ \frac{p_{\text{H}_2}}{p_{\text{H}_2}^{\text{ref}}} \exp \left( \frac{\alpha_e F (\eta_{\text{HOR}})}{RT} \right) - \left( \frac{a_{\text{HM}}^{\text{ref}}}{a_{\text{HM}}^{\text{ref}}} \right)^2 \exp \left( -\frac{\alpha_e F (\eta_{\text{HOR}})}{RT} \right) \right] \] (57)

where 1 and 2 denote the electron- and proton-conducting phases, respectively, and the reaction is almost always taken to be first order in hydrogen. Typically, the dependence on the activity of the proton(H)-membrane(M) complex is not shown since the electrolyte is a polymer of defined acid concentration (i.e., \( a_{\text{HM}}^{\text{ref}} = a_{\text{HM}}^{\text{ref}} \)). However, if one deals with contaminant ions, then equation 57 should be used as written. Also, it has recently been shown that the HOR may proceed with a different mechanism at low hydrogen concentrations; in this case, the kinetic equation is altered through the use of a surface adsorption term.\(^{96}\) Due to the choice of reference electrode, the reference potential and reversible potential are both equal to zero.

Unlike the facile HOR, the ORR is slow. Due to its sluggishness, the anodic part of the ORR is considered negligible and is dropped, resulting in the so-called Tafel approximation

\[ i_{\text{ORR}} = -i_{\text{ORR}}^0 \left[ \frac{p_{\text{O}_2}}{p_{\text{O}_2}^{\text{ref}}} \right] \left( \frac{a_{\text{HM}}^{\text{ref}}}{a_{\text{HM}}^{\text{ref}}} \right)^2 \exp \left( -\frac{\alpha_e F (\eta_{\text{ORR}})}{RT} \right) \] (58)

with a dependence on oxygen partial pressure, \( m_o \), of between 0.8 and 1\(^{97-100}\) and an Arrhenius temperature dependence for the exchange current density. For both the HOR and ORR, \( \alpha \) is typically taken to be equal to 1,\(^{97,98,101-103}\) however newer models use a value much closer to 0.5 for the ORR due to Pt-oxide formation.\(^{104,105}\)

The four-electron ORR involves oxide formation, which form at the potential range of the ORR (0.6 to 1.0 V) by water or gas-phase oxygen. These oxides can inhibit the ORR by blocking active Pt sites with chemisorbed surface oxygen. Typically, a constant Tafel slope for the ORR kinetics around 60 to 70 mV/decade is assumed over the cathode potential range relevant to PEFC operation. However, it has
been suggested by experiments that this approach has to be modified to account for the potential-dependent oxide coverage,\textsuperscript{104,106-108} especially for low catalyst loadings where poisoning has greater impacts due to the low catalyst-site number. To implement this coverage, a term is added to the ORR kinetic equation \((58)\),\textsuperscript{109}

\[
i_{\text{ORR}} = -i_{\text{ORR}}^{0} \left(1 - \Theta_{\text{PtO}}\right) \left(\frac{p_{\text{O}_2}}{p_{\text{O}_2}^{\text{ref}}}\right)^{m_0} a_{\text{HM}}^4 \exp\left(-\frac{\alpha_{a} F}{R T} \left(\eta_{\text{ORR}} - \frac{\eta_{\text{ORR}}^{\text{ref}}}{R T}\right)\right) \exp\left(-\frac{\omega \Theta_{\text{PtO}}}{R T}\right) \tag{59}\]

where \(\Theta_{\text{PtO}}\) is the coverage of Pt oxide; it should be noted that several oxides can exist and here PtO is taken as an example, and \(\omega\) is the energy parameter for oxide adsorption. There are various methods to calculate PtO using kinetic equations, such as\textsuperscript{110,111}

\[
\Theta_{\text{PtO}} = \frac{\exp\left[\frac{\alpha_{a} F}{R T} - \eta_{\text{PtO}}\right]}{\exp\left[\frac{\alpha_{a} F}{R T} - \eta_{\text{PtO}}\right] + \exp\left[-\frac{\alpha_{a} F}{R T} - \eta_{\text{PtO}}\right]} \tag{60}\]

where

\[
\eta_{\text{PtO}} = \Phi_1 - \Phi_2 - U_{\text{PtO}} \tag{61}\]

and \(\eta_{\text{PtO}}\) and \(U_{\text{PtO}}\) are the Pt-oxide overpotential and equilibrium potential, respectively. Another approach is estimated it by experimental correlations.\textsuperscript{112} A discussion of detailed mechanisms and expressions is beyond the scope of this review, although it is touched on in the section on catalyst-layer durability as it pertains to Pt ion formation. Similarly, other reactions related to durability and their associated kinetic expressions (e.g., carbon corrosion) are discussed in that section.
Critical Issues in the Field

As mentioned above, the two critical issues in the field for transport modeling to address are multiphase flow and modeling of catalyst layers. Specifically, there is still a need to resolve, understand, and model liquid-water transport and interactions, especially at the GDL / gas-channel interface. In terms of catalyst layers, they are the most complex layer within the PEFC, being composed of all other components and requiring knowledge of a wide variety of physics that interacts on multiple scales. In addition, there is still a lack of experimental data on issues related specifically to the catalyst layer, which makes modeling them challenging. Below, we examine each of these critical areas in sequence. We will discuss the governing equations for these issues, while highlighting those areas that require improvement and understanding. While the discussion will primarily remain focused on the continuum, macroscale phenomena, other lengthscales and approaches will be mentioned, especially in how they might interact with the continuum approaches or provide insight and are areas for future research.

Multiphase flow

Multiphase flow refers to the simultaneous existence and movement of material in different phases. It is well known that liquid and vapor coexist within PEFCs, especially at lower temperatures (e.g., during start-up) and with humidified conditions wherein the ohmic losses through the membrane are minimized. While multiphase flow in PEFC components and cells has been investigated deeply over the last two decades with substantial progress occurring recently due to advances in diagnostics, microscale models, and visualization techniques as discussed in this review, there are still unresolved issues, especially with the various interfaces and within the PEFC porous media and flowfield channels.
Nowhere is multiphase flow treated by such a variety of methods as in the composite DM (as shown in Figure 3, owing to the importance of these composite layers in water management. They provide pathways to disperse reactant fuel, oxidant, heat, and electrons while removing product water. A DM has to be hydrophilic enough to wick out the water and hydrophobic enough to not fill with liquid water and “flood” and block the reactant gas from reaching the catalyst site as mentioned above. This seemingly competing objective is met by partial treatment of the naturally mixed wettable layers with hydrophobic PTFE. Water produced at the cathode and water transported across the membrane is wicked out of the cell by capillary effects including perhaps transport through cracks and preferential pathways. A MPL serves to protect the membrane from being penetrated by the carbon fibers of the macroporous GDL, as well as provide discrete locations of water injection into the GDL.\textsuperscript{113-116} This decreases the water accumulation near the cathode and hence decreases oxygen mass-transport resistance.

In this section, we discuss the key approaches towards modeling multiphase flow, especially in DM. Of course, there are always caveats and simplifications that must be made to model the macroscale transport. Thus, the specific pore structure is often considered only in a statistical sense, local
equilibrium among phases is often assumed, and the effective properties, which are often measured for the entire layer, are applied locally and assumed to remain valid. We will however discuss some more microscopic modeling and how one can reconstruct PEFC porous media computationally. Next, we explore recent work on issues related to specific interfacial phenomena, contact resistances, and correlating the channel conditions to the droplets and water removal. Such effects could dominate the overall response of the cell and serve as boundary conditions or even as discrete interphase regions. Finally, multiphase flow also encompasses water freezing and melting, and the kinetics of such processes are reviewed.

Incorporation of multiphase phenomena

Modeling the PEFC porous media requires descriptions of the fluxes in the gas and liquid phases, interrelationships among those phases, as well as electron and heat transport. Traditional equations including Stefan-Maxwell diffusion (equation 18), Darcy’s law for momentum (equation 42), and Ohm’s law for electron conduction (equation 32) are typically used, where most of the effective transport properties of the various layers have been measured experimentally, or perhaps modeled by microscopic methods. Effective properties are required to account for the microscopic heterogeneity of the porous structures. This is accomplished by volume averaging all the relevant properties and system variables for transport within the porous domain,

\[ \psi_k^{\text{eff}} = \frac{\varepsilon_k}{\tau_k} \psi_k \]  

(62)

where \( \psi_k \) represents any property in the phase \( k \) and \( \varepsilon_k \) and \( \tau_k \) are the porosity and tortuosity of phase \( k \), respectively. As an example, the effective gas-phase diffusivities are both a function of the bulk porosity, \( \varepsilon_o \), and the saturation, \( S \), or the liquid volume fraction of the pore space,
\[ D_{i,j}^{\text{eff}} = \frac{\varepsilon G}{\tau G} D_{i,j} = \frac{\varepsilon_o (1 - S)}{\tau G} D_{i,j} = \varepsilon_o (1 - S)^n D_{i,j} \]  

(63)

The power-law exponents have been shown to have values of around 3 for typical fibrous GDLs,\(^{41,43,117,118}\)

\[ \frac{D_{i,j}^{\text{eff}}}{D_{i,j}} = \varepsilon_o^{3.6} (1 - S)^3 \]  

(64)

and are anisotropic with the in-plane value being several times larger than the through-plane one, which also agrees with microscopic modeling results.\(^{119,120}\)

For the bulk movement and convection of the gas phase, Darcy’s law and the mass-averaged velocity (equations 42 and 14, respectively) are used with an effective permeability that is comprised of the absolute (measured) permeability and a relatively permeability owing to the impact of liquid

\[ k_k = k_{r,k} k_{\text{sat}} \]  

(65)

where \( k_{r,k} \) is the relative permeability for phase \( k \) and is often given by a power-law dependence on the saturation.\(^ {41,121}\) The above equations are used along with the mass balance (equation 11) to describe the gas-phase transport.

While one can treat the liquid water as a mist or fog flow (i.e., it has a defined volume fraction but moves with the same superficial velocity of the gas), it is more appropriate to use separate equations for the liquid. This treatment is often of the form of Darcy’s law (equation 42), which, in flux form is

\[ \mathbf{N}_{w,l} = -\frac{k_L}{\bar{V}_w \mu} \nabla P_w \]  

(66)

where \( \bar{V}_w \) is the molar volume of water. One can also add a second derivative to the above equation such that a no-slip condition can be met at the pore surfaces (i.e., Brinkman equation).\(^ {122}\)
In terms of heat transport, for DM, the thermal balance turns mainly into heat conduction due to the high thermal conductivity compared to convective fluxes. Although no electrochemical reactions are occurring within the DM, there are still phase-change reactions that can consume/generate a considerable amount of heat, e.g.,

$$r_{\text{evap}} = k_{\text{evap}} a_{G,L} (p_w - p_{w}^{\text{vap}})$$  \hspace{1cm} (67)

where $k_{\text{evap}}$ is the reaction rate constant. In typical PEFC porous media, the area between the phases ($a_{G,L}$) and the reaction rate constant are normally assumed sufficiently high that one can use equilibrium between the liquid and vapor phases. The reaction source terms must be included in the overall heat balance (equation 50), and it should be noted that effective properties are again required to be used in the governing equation, where recent studies have shown the dependence of the anisotropic effective thermal conductivity on water saturation. One must also be aware of contact resistances and interfacial issues to determine the boundary conditions as discussed in the next section.

Finally, there are also modeling methodologies that assume equilibrium among the gas and liquid phases and try to recast the transport equations in order to be computationally more efficient or to converge easier; a prime example is the multiphase mixture model. In this analysis, although both liquid and vapor phases move simultaneously, they move at different velocities. This difference leads to a drag on either phase. The liquid-phase velocity can be calculated using

$$v_L = \frac{\lambda_L}{\rho_L} \frac{p_m}{\rho_m} v_m + \frac{k_{\lambda,L}(1-\lambda_L)}{\varepsilon_o \rho_L v_m} \left[ \nabla (p_L - p_G) + (\rho_L - \rho_G) \mathbf{g} \right]$$  \hspace{1cm} (68)

where the subscripts $m$ stands for the mixture, $\rho_i$ and $v_i$ are the density and kinematic viscosity of phase $k$, respectively, and $\lambda_L$ is the relative mobility of the liquid phase which is defined as,
A similar equation can be derived for the gas-phase velocity, which is then used to get the pressure drop in the gas phase while the Stefan-Maxwell equations (18) govern the diffusive transport of the gas species. In equation 68, the first term represents a convection term, and the second comes from a mass flux of water that can be broken down as flow due to capillary phenomena and flow due to interfacial drag between the phases. The velocity of the mixture is basically determined from Darcy's law using the properties of the mixture. While the use of the multiphase-mixture model does speed computational time and decreases computational cost, problems can arise if the equations are not averaged correctly. In addition, it does not track interfaces rigorously and is typically not seen as a net benefit for most PEFC models since they are not limited computationally, with an exception perhaps being 3-D models.

**Liquid/vapor/heat interactions**

Equations 66, 63, and 65 clearly show that there is an impact of the liquid- and gas-phase volume fractions on the transport of each other through the various effective transport properties. The key in calculating these relationships is determining an expression for the way in which the saturation varies with the independent driving force, namely pressure. From a continuum perspective, in a capillary-dominated system, these are related through the capillary pressure:

\[
S = f(p_c)
\]

\[
p_c = p_L - p_G = -\frac{2\gamma \cos \theta}{r}
\]

(70)

where \( \gamma \) is the surface tension of water, \( r \) is the pore radius, and \( \theta \) is the internal contact angle that a drop of water forms with a solid. This definition is based on how liquid water wets the material; hence, for a hydrophilic pore, the contact angle is \( 0^\circ \leq \theta < 90^\circ \), and, for a hydrophobic one, it is \( 90^\circ < \theta \leq 180^\circ \).
The capillary pressure can also impact the saturation vapor pressure, which should be corrected for pore effects by the Kelvin equation,

\[ p_{vap}^0 = p_{vap}^0 \exp \left( \frac{p_c V_w}{RT} \right) \]  

(71)

where \( p_{vap}^0 \) is the uncorrected (planar) vapor pressure of water and is a function of temperature.

The functional form of the saturation dependence on capillary pressure can be measured\textsuperscript{128-133} or derived using various simplistic models\textsuperscript{113} or more complicated pore-network and other models as discussed in the next section. Figure 4 displays exemplary capillary-pressure – saturation relationships. From the figure, one sees that addition of a hydrophobic (i.e., Teflon) treatment causes the curve to shift towards higher capillary pressures (i.e., more hydrophobic). Also, the gas-diffusion layer exhibits an intermediate wettability where there is a hysteresis between imbibition and drainage that spans \( p_c = 0 \). However, this hysteresis is not as important since PEFCs operate on the imbibition curve during operation (unless drying is being done) since water is always being injected from the catalyst layer to the gas channel. Similarly, the full curve is not expected to occur because the high permeability of tradition gas-diffusion layers means that once breakthrough and a dominant pore pathway has formed, it is sufficient for removing the liquid water assuming that the ribs do not block too much of the exit pathway. Also shown in the figure is a curve for a catalyst layer, which, due to its small pores, has a wider range of pressures. It is also measured to be more hydrophilic than a GDL, especially if it contains cracks.\textsuperscript{131}
Figure 4 – Capillary-pressure – saturation relationships for a crack-free catalyst layer,$^{131}$ and two SGL GDLs with 0 (blue) and 5 wt-% (red) Teflon.$^{113}$

For use in models, the measured capillary-pressure – saturation relationship is often fit to a function (e.g., hyperbolic tangent) or to a Leverett $J$-function,$^{119,127}$

$$P_C = \gamma \cos \theta \left( \frac{\epsilon_o}{k_{sat}} \right)^{0.5} J_S$$  \hspace{1cm} (72)

However, such a treatment is not rigorously defined for fibrous media and it has limited applicability in terms of predictability in terms of the properties in the above equation.

A less empirical way to treat the capillary-pressure data is to fit the curve using the separately measured pore-size distribution and a contact-angle distribution.$^{113}$
\[ S^* = \int_0^{\theta_\text{o,k}} \Psi(\theta) \left( \sum_k \frac{f_{r,k}}{2} \left[ 1 + \text{erf} \left( \frac{\ln r_c - \ln r_{o,k}}{s_k \sqrt{2}} \right) \right] \right) d\theta + \int_0^{180} \Psi(\theta) \left( \sum_k \frac{f_{r,k}}{2} \left[ 1 - \text{erf} \left( \frac{\ln r_c - \ln r_{o,k}}{s_k \sqrt{2}} \right) \right] \right) d\theta \]

\[ = \sum_{n=0}^{90} f_{\theta,n} \left\{ \frac{1}{\sigma_n \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{\theta - \theta_{o,n}}{\sigma_n} \right)^2 \right] \right\} \sum_k \frac{f_{r,k}}{2} \left[ 1 + \text{erf} \left( \frac{\ln \left( \frac{-2\gamma \cos \theta}{p_c} \right) - \ln r_{o,k}}{s_k \sqrt{2}} \right) \right] d\theta \]  

\[ + \int_0^{180} \sum_k \ldots \]  

where \( r_{o,k} \) and \( s_k \) are the characteristic pore size and spread of distribution \( k \), respectively, and \( f_{r,k} \) is the fraction of the total distribution made up of distribution \( k \), where the \( f_{r,k} \)'s sum to unity; the contact angle distribution is given by

\[ \Psi(\theta) = \sum_n f_{\theta,n} \left\{ \frac{1}{\sigma_n \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{\theta - \theta_{o,n}}{\sigma_n} \right)^2 \right] \right\} \]  

where \( \theta_{o,n} \) and \( \sigma_n \) are the characteristic contact angle and deviation of distribution \( n \). The integration in equation 73 is done with respect to the critical radius as determined from equation 70,

\[ r_c = -\frac{2\gamma \cos \theta}{p_c} \]  

which is a function of hydrophobicity and thus the integral is separated into terms representing hydrophobic and hydrophilic contact angles since the critical angle approaches infinity at a zero capillary pressure. Finally, one can also incorporate the residual saturation, \( S_r \), as shown in Figure 4

\[ S = S_r + S^* \left( 1 - S_r \right) \]  

Using the above integration approach, one can also determine expressions for the relative permeability of the liquid and gas of
\[ k_{r,L} = S_e^2 \int_{0}^{90} \Psi(\theta) \left( \frac{1}{k} \sum_{r} f_{r,k} r_{o,k}^2 \exp\left[2s_k^2\left(1 + \text{erf}\left(\frac{\ln r_c - \ln r_{o,k}}{s_k \sqrt{2} - s_k \sqrt{2}}\right)\right)\right] \right) \, d\theta + \int_{90}^{180} \Psi(\theta) \left( \frac{1}{k} \sum_{r} f_{r,k} r_{o,k}^2 \exp\left[2s_k^2\left(1 - \text{erf}\left(\frac{\ln r_c - \ln r_{o,k}}{s_k \sqrt{2} - s_k \sqrt{2}}\right)\right)\right] \right) \, d\theta \] (77)
measured or typically predicted for the entire layer. The validity of this assumption still remains an open question, especially since it is known that DM structures are not spatially homogeneous.

Although the liquid and gas phases are related through transport properties, they also have an effect on each other’s fluxes through heat transport and phase-change-induced (PCI) flow. In this fashion, the liquid water is near equilibrated with water vapor and the temperature distribution induces a water vapor-pressure gradient. The water is transported along that gradient and condenses and gives off heat at the gas channel or cooler flowfield rib as shown in Figure 5. Such an effect can be shown to be able to move all of the produced water when operating above temperatures of 60°C or so with typical component properties. In this fashion, the produced water is transported through the cell components in the vapor phase and flooding concerns will be minimal and result mainly from condensation in the flowfield channels or at the GDL / flowfield-land interface. Thus, PCI flow is the dominant mode of water removal at higher temperatures, and may even cause the cell water content to decrease at higher current densities because the heat generated outpaces the water generation. However, while the water-removal characteristics are a benefit of PCI flow, the net flux of water vapor is now out of the cell, which can results in the gas-phase velocity also being out of the cell (see equation 14). In either case, the movement of the water vapor from the catalyst layer to the gas channel due to PCI flow represents a mass-transfer limitation in terms of getting oxygen to the catalyst layer since it now must diffuse against that flux. Finally, PCI flow also results in substantial heat removal from the hotter catalyst layer as well.
Figure 5. Schematic representation of phase-change-induced (PCI) flow.

Microscale modeling and reconstruction

Within the area of numerical simulation for the performance and durability of PEFCs, there has always been a great interest and need to understand the linkage between the properties of the component materials in the MEA and the performance of the unit cell and stack. As research progressed and an increasing level of detail was paid to the fabrication, engineering, and effect of the individual component properties on the overall PEFC performance and durability, there began to emerge attempts to resolve the morphology of the components with ever increasing levels of resolution and detail. These morphological-based simulations are typically ascribed with the label of being microstructural simulations and are related significantly to transport and multiphase aspects.

For each component within a PEFC, a relevant length scale can be considered dependent on the physical process being considered and the morphology of the component itself. In regards to this, one typically finds that the analysis of the PEFC porous media (i.e., GDLs, MPLs, catalyst layers) are accomplished using similar methodologies, while analysis of the polymeric membrane, for example, has traditionally lent itself more towards the application of molecular dynamics-based approaches. While the latter is outside the scope of this review, some remarks should be made on the former. Morphological-based models offer the potential to investigate explicitly the relationship between the
porous media and the specific transport property generally with consideration given for the constituent material properties, spatial distributions, and statistical variations that may originate within manufacturing processes. In order to make these linkages, however, a virtual method of representation of the component structure is required.

Porous media within PEFCs are generally accepted to fall into the category of random, heterogeneous porous media. It was from the aerospace, metallurgy, and energy industries that some of the earliest attempts at morphological analysis and numerical reconstructions are found. Typically, there are two main methodologies employed in generating a virtual morphology with the first being a stochastic-based reconstruction method in which a statistical distributions, randomness, and a series of rule-sets are used as a basis to create a “representative” morphology. The second method, image-based reconstruction, employs experimental diagnostic imaging, such as SEM, TEM, or x-ray computed tomography (XCT), combined with a numerical processing step that converts the images to a virtual structure thus translating the original experimental images into a morphology that can be used for numerical analysis.

For the latter approach, the porous medium of interest in imaged via one of several possible methods and the resulting imaging data is then processed and translated into a virtual structure. This can be accomplished, for example, by extracting images from the center lines of single fiber from the resultant images of GDLs. These center lines are then used in conjunction with a stochastic algorithm to reconnect parts of the center lines in an attempt to preserve the curvature of the fibers. The limitation of the rendered pore space and features within the domain remain the limit of the spatial resolution of the imaging technique itself as well as the ability to determine the distinct phases (e.g., Teflon and carbon and sometimes water are not easy to distinguish with x-rays).
For stochastic reconstruction, the virtual generation of the microstructures relies on the use of a random number generator, statistical distribution of geometric information relating to the constituent materials that comprise the media, and a series of rule-sets. Typically these rule-sets are an attempt to impose physical constraints on the placement algorithms in order to make the creation of the domain more tractable, such as constraining fiber size, overlap, interactions, etc. The constraints placed on the invariance of domain properties in the non-through-thickness directions are directly related to the overall numerical intensity of the final simulations as constraining the morphological model to as small a sample as possible dramatically increases the speed of the simulations. However, the main criteria in deciding this aspect should relate primarily to the size of the morphological domain relative to the transport coefficient of interest, wherein the transport coefficient does not vary if the domain size changes (i.e., it is a representative volume element). An example of a reconstruction of a GDL is shown in Figure 6, where one can clearly delineate the fibrous structure.
Figure 6 – Schematic of a computationally reconstructed carbon-fiber paper147

For all of these simulations, it should be noted that predicting a volume-averaged coefficient for use in macroscopic models necessitates conducting simulations enough times to get representative structures and values. Thus, the use of a quality random number generator is paramount in generating an appropriate numerical representation, where it has been suggested that the use of 25 samples is sufficient to represent an appropriate sample set given the stochastic nature of the experimental geometry data.148 The need for the larger sample set is especially the case since validation of the microstructures by imaging and other methods (e.g., porosimetry) are typically through general, statistical information,149 of which various microstructures may still agree with the data. Ultimately, it is best to compare aspects that relate directly to the distribution of material through the sample but on a statistical or probabilistic basis thus the use of both imaging and measurable characteristics are ideal.

Due to the increasing amount of structural characterization and the virtual microstructures, it is now possible to model transport through the DM using direct simulations.139,145,150,151 These models can provide the critical effective properties needed for the cell-level models, however, they require detailed information on internal surfaces and chemical information (e.g., wettability), which, as described above, is not always readily available. In addition, they are still numerically intense (especially for the statistical number of simulations required). For example, Zamel et al. discuss that the mesh in their simulations ranges from 135,000 to 2.3 million grid points with the number depending on the total porosity of the structure.145 Similarly, Lattice-Boltzmann methods may likewise be used but require the same kind of detailed chemical and structural information as well as knowledge of the underlying forces acting on and between the various fluids and components.152-162 They have also been shown to not necessarily provide
significantly more insight, especially for use in macroscopic simulations and with typical GDL saturations around 0.1 to 0.3.\textsuperscript{163} A detailed description of such methods is beyond the scope of this review due to the above mentioned issues.

A large of amount of research and analysis has been done to improve the capability in the morphological modelling area in the last decade. Domain generation has become sufficiently more sophisticated with techniques being developed to generate stochastically not just fiber arrangements but full DM structures including Teflon, binder, and MPLs.\textsuperscript{164} Thus, such models are valuable areas for continued research and development. In addition, experimental techniques in terms of characterization of the various effective properties are also increasing in scope and complexity as discussed in a later section. However, as mentioned in the previous section, most of these experiments remain valid for only the entire layer, and not as a function of DM thickness. Thus, to be used in situations where there are gradients and flow, one needs to assume local equilibrium and that a given volume element is representative of the entire domain; such an assumption remains unproven. Overall, there is a need to understand the physics within the DM but without necessitating the determination of a substantial set of unknown parameters. One such treatment that is currently striking such a balance is pore-network modeling.

\textit{Pore-network modeling}

A promising treatment that satisfies those requirements is that of pore-network modeling.\textsuperscript{91,92} Such an approach allows one to model the pore-length scale, and therefore account for the capillary-driven processes, thermal processes, and water distribution. However, it is still too computationally costly to account for all of the coupled physics along with the rest of cell components. Therefore, it is valuable for
use in a multiscaling approach wherein the pore-scale models yield the functional relationships required for the more macroscopic complete-cell models (e.g., saturation, effective properties, etc.). Pore-network modeling allows for consideration of more physical and chemical properties than the macroscopic approaches described in the preceding section.

A pore-network model utilizes a simplified description of the pore space within the DM. Thus, one idealizes the geometry in terms of pores and interconnections (nodes) as shown in Figure 7. The throat lengths and sizes are taken from analysis of either theoretical or actual (e.g., XCT) images of the DM that are digitized and analyzed using methods and approaches mentioned above in the previous section. The generated network is validated by comparison of calculated and measured parameters including the pore- and throat-size distribution data as well as measurements such as the capillary pressure – saturation relationship. Water flow and distribution within the generated network (Figure 7) is solved by a stepwise fashion from one point to another, and thus is independent of the real-space discretization grid (i.e., it only depends on the network).

Figure 7 – Schematic of a pore-network-model solution.
For modeling transport, the same governing equation and multi-phase phenomena described in the above sections remain valid. For example, for liquid-water imbibition, the model examines each intersection or node where the water travels based on the local pressure and pore properties. The volumetric flowrate of water in a cylindrical pore of radius $r_{ij}$ between nodes $i$ and $j$ (see Figure 7) is governed by Poiseuille flow

$$q_{w,\text{pore}} = A_{\text{pore}} \cdot v_{w,\text{pore}} = \frac{\pi r_{ij}^4}{8 \mu_{ij}^{\text{eff}} l} \left( \Delta p_{ij} - p_{c_{ij}} \right)$$  \hspace{1cm} (81)

where $\Delta p_{ij}$ is the pressure acting across the pore, $l$ is the pore length, and $p_{c_{ij}}$ is the capillary pressure in the pore when multiple phases are present. The volumetric flowrate exists only when $\Delta p_{ij} > p_{c_{ij}}$. The effective viscosity within a pore, $\mu_{ij}^{\text{eff}}$ is a function of the fluid position inside the pore, $x_{ij}$, the non-wetting (injected) fluid viscosity, $\mu_{nw}$, and the wetting (displaced) fluid viscosity, $\mu_{w}$. The effective viscosity is modeled to provide a smooth transition between the wetting and non-wetting viscosities while a pore is neither completely filled nor empty. The capillary pressure is also modeled as a function of the fluid position within each pore. It is calculated similar to equation 70 but where either the average radius of the intersecting pores at each node or the radius of a given pore is used depending on where the water meniscus exists,

$$p_{c_{ij}} = \gamma \cos \theta \left[ \left( \frac{1 - r_i}{2 r_i} - \frac{r_j}{2 r_j} \right) \frac{1 - \cos \left( \frac{2 \pi x_{ij}}{l} \right)}{r_{ij}} + \frac{1 + \cos \left( \frac{\pi x_{ij}}{l} \right)}{r_i} + \frac{1 - \cos \left( \frac{\pi x_{ij}}{l} \right)}{r_j} \right]$$  \hspace{1cm} (82)
where \( \bar{r}_i \) and \( \bar{r}_j \) are the average pore radius around node \( i \) and \( j \), respectively. The capillary pressure is zero when the pore is filled with only one fluid. Conservation of mass requires that the flowrate balance at each node for every simulation step, thus from equation 81 one gets

\[
\sum_j \frac{r_{ij}^4}{\mu_{ij}^{\text{eff}}} (\Delta p_{ij} - p_{c_{ij}}) = 0
\]  

(83)

where the summation is over all of the pores connecting to the node (normally 4). The unknown pressure gradient, \( \Delta p_{ij} \), is solved through the equation above. In addition to the pore sizes and lengths, one also needs the pore contact angle and fluid properties.

A sample output of a pore-network model is shown in Figure 8.\textsuperscript{165} As can be seen, the model can show the distribution of the water, and in the top figure one can see that a dominant pore pathway has formed. The simulations are stochastic, similar to the transport of water in the DM, and thus enough samples are required for understanding as discussed above. Such distributions can also provide direct insight and help in terms of understanding the impact of the water network on the exterior interfaces as
discussed in sections below. The distributions can then be used in other transport simulations to predict the gas-phase tortuosity and effective diffusion coefficients, which are needed for the macroscopic modeling of the gas phase. Currently, the pore-network models do not yet contain all of the physics, although advancements such as including phase change and coupled thermal effects are providing promising results. Work is also needed in understanding and incorporating thickness-dependent structural heterogeneities and in linking these models with boundary conditions defined by interfacial interactions with the other PEFC components.

**Interfaces**

The impact of interfacial regions in the PEFC has so far been commonly neglected in classical full-cell multiphase modeling efforts. Traditionally, the interface between two layers or the channel wall and DM has been neglected or simply treated as an infinitely thin barrier with discrete property values on either side and no co-mingling of surfaces and properties. However, more recent modelling and experimental work has demonstrated the importance of these interfaces in water storage, transport, and durability. Also, the interfacial conditions typically drive the transport phenomena and are key in determining the boundary conditions for the various models. The specific interfaces of interest discussed in this section include the DM with the catalyst layer, the internal boundary between the MPL and GDL, and the interactions and interface between the GDL and flowfield, which is perhaps the most critical in terms of controlling transport phenomena.

Classical macroscopic models only consider a bulk contact angle, pore size, tortuosity, porosity, and thermal conductivity in calculation of liquid transport with bulk capillary-transport functions, and ignore any interfacial regions, or capillary action formed by the channel wall interface with the diffusion medium. However, evidence definitively shows that the performance is quite different when the
interfacial conditions above are changed, even if the normal multiphase parameters listed are identical. Clearly, there is much more to study in order to develop the next-generation of optimized materials and designs with advanced multiphase models incorporating these effects.

*Catalyst layer / microporous layer*

The vast majority of literature treats this interface as a perfectly mated, parallel interface. The reality is, however, that the two mating surfaces are not perfectly smooth or straight as shown in Figure 9, and a mismatch between the elasticity and surface roughness of the interfaces results in small gaps and non-contact zones between the two surfaces, even under elastic compression.\(^{171}\) Additionally, in conventional channel/land architectures, the transmitted compression pressure under the channels is much less than under the lands, which can also result in a different interface compression condition. The net effect of this interfacial roughness mismatch between the MPL and catalyst layer is a zone of imperfect contact with microscopic gaps as shown in Figure 10. Figure 10b was formulated by application of a compressed-surface-interaction model and experimental topographical scans of a commercial MPL and catalyst surface.\(^{172}\) Based on these findings,\(^{171-177}\) the potential interfacial gap storage volume can be significant, and depends directly on the elasticity, smoothness of the mating surface (and if there is a mismatch in roughness) and compression pressure. Thus, the worst condition is under a channel with near zero overburden (compression) pressure, with a cracked, stiff MPL with much greater surface roughness than the CL. Under these conditions, the local gaps at the CL|MPL interface can be significant and can impact the transport phenomena in the cell.
Figure 9. Optical micrographs and 3D surface heights for a) MPL with cracks, b) MPL without cracks and c) CL (Images from reference 174).

Figure 10. a) Schematic of compressed rough surfaces under compression and b) Calculated and c) computed 3-D interfacial compression based on real MPL and CL surfaces (Images from references 174 and 171).
The additional gaps created between the surfaces will affect the electronic transport across the interfaces. However, unless the gaps are exceptionally large, there will be no significant impact on the measured performance due to ohmic losses between a mismatched and well-matched interface. The key controlling parameters in the presence of additional losses caused by a gap are the in-plane conductivity of the mating layers, the width of the gap, and thickness of the interface. Bajpai et al.\textsuperscript{173} computed losses along the MPL interface as a function of gap width and thickness and found that for normal conductivities, ohmic losses are minor until a dimensionless gap width of around 10 (see Figure 11).\textsuperscript{173,174,176} This dimensionless ratio is also relevant for other modes of transport. Thus, interfacial gaps that would have significant ohmic implications are not likely in a normal catalyst layer / MPL interface, without some blistering or delamination caused by ice damage or other phenomena.

Similar to ohmic transport, the thermal disruption from the small gaps was calculated to be minimal for normally distributed surface roughness related discontinuities under compression.\textsuperscript{173,176} Larger disruptions, or low contact pressure, result in local hot spots (if gas filled) or cooler spots (if liquid filled).
Figure 11. Calculated increase in cathode ohmic and kinetic losses as a function of gap width to thickness ratio in the MPL (Image from ref 173).

Small local gaps and interfacial contact gap volumes are locations where liquid-water accumulation is favored due to low local capillary pressure. Thus, these reservoir volumes will fill and pool first and will subsequently be the last to drain or evaporate upon shutdown. Similar to the ohmic and heat-transfer considerations, isolated small volumes of liquid are not predicted to be a major factor in the performance, as the effective gas-phase transport distances around the volume are not significantly larger than the normal path lengths at normal saturation levels. However, as shown in Figure 12, they can contain a non-negligible amount of the total liquid water.176 Of greater importance is the possible existence of a liquid film along the interface, which hinders the gas transport to the catalytic sites. Direct evidence of the presence of water films along the catalyst layer / MPL interface under the channel (where overburden pressure is lower) is shown in Figure 13, in terms of an ice-lens growth following freeze/thaw cycling.178-181 As shown in Figure 13b, the high-frequency resistance of the cell was increased, indicating a significantly large ice lens was present in agreement with the analysis of Figure 11.
Figure 12. Predicted distribution of liquid water in the cathode, with interfacial voids (Image from ref 176).

Figure 13. (a) Evidence of interfacial ice lens formation and delamination after 100 freeze-thaw cycles (b) measured high-frequency resistance showing evidence of interfacial delamination impact under extreme conditions (images from reference 179).

As mentioned, the MPL provides specific entry points into the GDL for liquid water. These pathways are nominally formed at cracks. From a gas-phase heat- and mass-transport perspective, these cracks should have little effect. Thus, impact from the cracks is expected to be a result of changes in liquid-water motion and accumulation. Similarly, access of liquid-water to the cracks requires sufficient in-plane transport in the catalyst layer or along the heterogeneous interface. Thus, the rough interface may provide for better water removal, but at the detriment of forming a film and block gas transport. Similarly, pooling in these gaps can also enhance low RH operation,\textsuperscript{137,138} but perhaps not higher RH operation if there is liquid condensing in the GDL and trying to move back into the catalyst layer.\textsuperscript{182} While the engineering of water pathways throughout the whole DM has been investigated experimentally,\textsuperscript{183-188} there is a lack of modeling studies. In addition, the tradeoffs of having interfacial gaps and their possible engineering along the interface is something that modeling can, but has yet to, address in a substantial fashion.
Microporous layer / gas-diffusion layer

The boundary between the GDL and MPL is arguably the least studied interface in a PEFC. Part of the issue is that neutron and other visualization techniques have so far failed to provide the sub-micrometer resolution needed to confirm various modeling approaches. However, it is known that this interface is not sharp (see Figure 3), and has been measured by x-ray tomography to have a standard deviation of more than 20 μm. The classic treatment of this boundary is again as infinitely thin with discrete media with different transport parameters on both sides, especially since the governing equations throughout both domains remain the same. The result of this assumption is the prediction of a “saturation-jump” between the MPL and DM, where the high hydrophobicity of the MPL and small pores results in a very low saturation in this layer that is discontinuous with the larger pore size, lower hydrophobicity GDL. In practice, however, neutron imaging and higher-resolution x-ray imaging has not shown this to be the case, although due to resolution and imaging limitations as well as the intermixing of the MPL / GDL components, the evidence is not conclusive.

Gas-diffusion layer / flowfield

The GDL / flowfield interface is the most widely studied of the interfaces, especially as it is critical for water management. Several of the effects described above such as PCI flow are directly related to this interface. In addition, the boundary condition at the channel / GDL interface controls much of the water retention in the cell and is of paramount importance in terms of understanding and predicting multiphase flow. This boundary condition is typically a specification of flux or concentration for water vapor and one of saturation or liquid pressure for liquid water. However, if one is using a two-phase model without a residual effective permeability, then setting saturation equal to zero could be
problematic in terms of convergence since this condition enforces the fact that all water must leave the GDL in the vapor phase since the effective permeability will go to zero. We believe that it is better to set the liquid pressure or capillary pressure, and ideally this pressure should be associated with the formation and existence of droplets on the GDL surface as described below.

The GDL / flowfield land interface is of greatest importance in electron\textsuperscript{192,193} and heat transport\textsuperscript{194}, as the contact resistance across this boundary can be highest here compared to other interfacial layers. In conventional channel/land architectures, the land is impermeable and is cooler than the GDL/ channel interface due to direct conduction to the flowfield plate. Thus, liquid tends to accumulate under the land by condensation,\textsuperscript{28,135,195} until removal via in-plane flow or evaporation. Figure 14 illustrates the process of under-land accumulation, followed by flow into the channels via capillary action up the channel walls into droplets if the channel wall is hydrophobic (Figure 14a) or film if hydrophilic (Figure 14b).\textsuperscript{196} The shape of the interface should also play an important role in the water removal, as a rounded land edge will tend to more efficiently draw water out of the GDL along the channel-wall interface than an abrupt corner will.
Figure 14. Schematic and neutron image of water accumulation and capillary motion for hydrophobic and hydrophilic channel-wall interfaces. (Image from ref \cite{196}).

The GDL/flowfield channel interface is important for water management as mentioned, as the gas must enter the GDL and water must be removed. As shown in Figure 13, this can be accomplished through capillary interaction at the lands. Alternatively, liquid water can be removed as droplets in the channel assuming conditions of multiphase flow exist (i.e., 100% RH). Once in the channel, the water moves either along the walls, as a mist, or stochastically by forming films that are periodically removed due to gas-pressure buildup.\cite{71,197,198} The key conditions for this boundary condition are being able to predict the surface coverage of water or droplets and the subsequent capillary pressure or liquid pressure as a function of material properties and operating conditions. This is something that has been researched both theoretically and experimentally, but not yet clarified and utilized, especially in PEFC modeling.
To understand water-droplet behavior, emergence, and detachment, and to provide more physical basis for modeling the interface, detailed droplet-specific studies have been accomplished. These studies have highlighted some of the fundamental issues and have examined liquid-water droplet and water removal from PEFC flowfield channels. It has been shown that droplets form periodically at specific locations and with an associated varying liquid pressure, which can be related to the breakthrough pressure and then subsequent growth and detachment phases as shown in Figure 15. Analyzing the 110 µm curve, the initial pressure signal is due to head from water traveling through system lines and filling void space between the GDL and injection port. At approximately $t = 700$ s, a steep climb begins due to water being forced into GDL pores; the slope is attributable to expansion of the injection system. At about $t = 1200$ s pressure reaches a maximum which is accompanied by droplet formation on the GDL surface. This maximum pressure is the breakthrough pressure, followed by a dramatic fall in signal as the pressure decays to Darcy resistance inside the GDL pores. Subsequent runs will then show a smaller rise between the breakthrough and Darcy pressures. This liquid pressure could be used as the boundary condition for the liquid transport equations (e.g., equation 42), but the fact that it is isolated and time varying makes its use complicated; more experimental and computational studies are needed.
Figure 15: Breakthrough pressures as a function of injection time (left) and as a function of PTFE loading in the GDL (right).²⁰⁵

In terms of in-situ testing, it has been shown that pressure forces can impact the water-removal rate by removing the droplets.¹⁹⁷,²⁰² High-resolution images have also been used to elucidate the dynamic behavior of liquid water on a GDL surface, including droplet coalescence, detachment by the gas core flow, and wicking onto hydrophilic channel walls (see Figure 13).¹⁹⁷,²⁰²,²⁰³ For example, droplet detachment as a function of gas velocity is shown in Figure 16.²⁰³ Real-time images of water transport through the GDL were also captured by Litster et al. using fluorescence microscopy, which allowed for pore-level detail.²⁰⁶ They observed the dynamic behavior of water traveling through specific pathways of the GDL and concluded that fingering was the primary mechanism by which water traverses the pore network.

\[ u_{\text{air}} = 0 \text{ m/s} \quad u_{\text{air}} = 8 \text{ m/s} \quad u_{\text{air}} = 11 \text{ m/s} \quad u_{\text{air}} = 13 \text{ m/s} \]
Figure 16. Sequence of surface distortion of a 1.0 mm droplet before detachment from GDL surface.\textsuperscript{203}

To understand the process of detachment, a force-balance approach can be used,

\[ F_a + F_p + F_s + F_g = 0 \]  

where \( F_a \) is the adhesion or surface-tension force, \( F_p \) is the pressure force, \( F_s \) is the shear force acting on the droplet, and \( F_g \) is the gravitational force, which is negligible for typical droplet sizes although can be more important for large slugs. The pressure and shear forces are readily calculated from correlations and flow equations through the channel and associated drag effects.\textsuperscript{199,200,207,208} For example, the shear force can be calculated based on Stokes flow past a sphere and the shape of the droplet,

\[ F_s = \left[ \frac{6\mu H \langle v \rangle}{(H - h)^2} \right] d^2 \]  

where \( d \) is the droplet diameter at its maximum, \( H \) is the channel height, \( \langle v \rangle \) is the average flow velocity in the channel under laminar flow,\textsuperscript{82} \( \mu \) is the fluid viscosity, and \( h \) is the droplet height.

However, the adhesion force, which represents the resistance force that a droplet needs to overcome to initiate motion along a surface, is not as straightforward. Typically, this has been correlated to the static contact angle and contact-angle hysteresis. However, since the GDL surfaces are rough, it does not provide an accurate estimation of the adhesion force between the liquid-water droplet and GDL surface. In addition, measurements based on the static contact and contact-angle hysteresis seem to be more qualitative than rigorously quantitative in terms of droplet mobility. A recent study shows
that the sliding-angle experiment provides a better measure of droplet mobility and detachment from a rough and porous surface, like GDLs. The sliding-angle experiment measures directly the sliding angles and adhesion forces for liquid-water droplets on GDL surfaces,

\[ F_a = \frac{\rho V g \sin \theta_s}{\pi d_w} \]  

where \( \rho \) is the water density, \( V \) is the droplet volume, \( g \) is the gravitational acceleration constant, \( \theta_s \) is the sliding angle, and \( d_w \) is the wetted diameter, which is the diameter of the contact area between the liquid-water droplet and GDL surface. Experiments have also shown that a bottom injection, as in a PEFC, demonstrates a much larger adhesion force due to the underlying liquid network than a top placement, which many studies use.

As seen above, the key issues for the GDL / flowfield interface are in understanding the water removal into the channel. As discussed, multiple mechanisms including capillary interactions on the channel walls, condensation due to PCI flow, and droplet removal due to force balances can all contribute to the water removal. In addition, changes in liquid pressure due to droplet emergence and breakoff impacts the overall liquid holdup in the cell. Seeing as these effects are stochastic and complex in nature, delineation of the correct interface conditions for models has yet to be obtained and validated.

**Ice formation**

In the preceding sections, we focused on multiphase aspects in terms of liquid and vapor. However, in both automotive and stationary applications, PEFCs must permit rapid startup with minimal energy from subfreezing temperatures (i.e., cold-start), where water can solidify to form ice in the MEA. This freezing can severely inhibit cell performance and often results in cell failure. In recent years,
several cold-start targets have been established by the Department of Energy. Two key targets are that the PEFC must be able to start unassisted from $-40^\circ C$, and reach 50 % net power within 30 s from $-20^\circ C$. Despite significant attention, successful cold-start from $T < -20^\circ C$ remains a challenge. Achieving such a startup is difficult in practice due to potential flooding, sluggish reaction kinetics, durability loss, and importantly, rapid ice crystallization that is counteracted by the heating of the cell due to the reaction waste heat and ohmic heating. Typically, the cell potential decays rapidly at low temperatures and/or high current densities due to ice formation at the reactive area of the cathode. The traditional method to provide successful cold-start is to purge the system of water on shutdown, thus providing a sink in the membrane for the water production during startup and allow for the heating of the stack before irreversible flooding and ice formation occur. Such a strategy seems to work above $-20^\circ C$, but not at lower temperatures, due to issues discussed below; in addition, one prefers to develop and understand a possible optimal and material solution to cold-start.

Over the past decade, several numerical continuum cold-start models have been developed. To counter the difficulties associated with cold-start, models emphasize both procedural strategies and materials design. For example, Balliet et al. recommend higher potentials during startup to optimize performance from $-20^\circ C$ as well as increased water capacity or reservoirs (e.g., increased porosity). Numerous studies have also examined the stack-level thermal response during cold-start. However, relatively few studies model the water and ice within the PEFC. Early models assume that product water vapor instantaneously solidifies when the vapor partial pressure exceeds the saturation value. As a result, they do not account for liquid water explicitly within the PEFC.

More recently, Jiao et al. and Balliet et al. extended cold-start models to include vapor, liquid, and solid phases of water within the PEFC. The equilibrium freezing point of ice within the GDL, catalyst
layers, and PEM is based on a characteristic pore size using the thermodynamic Gibbs-Thomson equation,

\[ T_{FPD} = \frac{-2V_{ice} \gamma T_m}{r \Delta H_f} \]  

(86)

where \( T_{FPD} \) is the amount of freezing-point depression, \( V_{ice} \) is the molar volume of ice, \( \gamma \) is the surface tension of the ice-liquid interface, \( T_m \) is the melting (freezing) temperature for bulk water, \( \Delta H_f \) is the heat of fusion of ice, and \( r \) is the pore radius. The amount of freezing-point depression in a given pore is primarily a function of pore radius – smaller pores tend to freeze at a lower temperature due to the shift in chemical potential. Because real media have distributions of pore radii, the fraction of unfrozen water versus temperature is generally a continuum below 0°C. The overall rate of ice formation is then expressed as a linear rate equation

\[ R_f = k_f \left( S_L - S_L\left(T_{FPD}\right) \right) \]  

(87)

where the equilibrium liquid saturation, \( S_L\left(T_{FPD}\right) \), is a function of pore properties and is derived from a thermodynamic relationship.81,224

This thermodynamic-based freezing circumvents the use of ice-crystallization rate expressions, since at the time, none were available for PEFC-porous media. Furthermore, in recent years, in-situ visualization and detection of ice formation within PEFC-porous media has progressed.213,225-227 In all cases, generated water was observed initially in the subcooled state, particularly between –2 and –20°C. Although water did not freeze immediately, the mechanisms and kinetics of ice crystallization were not investigated.
Recently, Dursch et al. measured the kinetics of ice nucleation and growth in traditional GDLs and catalyst layers using differential scanning calorimetry.\textsuperscript{210,228,229} For the first time, they demonstrated that the kinetic-based approach shows a significant departure from the commonly-used Gibbs-Thomson equation. In all cases, ice crystallization was preceded by a limiting stochastic induction time, $\tau_i$, that depends strongly on both temperature and material properties.\textsuperscript{210,228,229} From these kinetic data, they developed validated ice-crystallization rate expressions to aid numerical continuum cold-start models. The rate expressions are based on the Johnson-Mehl-Avraham-Kolmogorov (JMAK) formalism of the form\textsuperscript{210,228,229}

$$R_I = k \left(\frac{T_c}{T}\right)^2 \left[1 - \phi - \ln(1 - \phi)\right]^{\frac{3}{2}}$$ \quad \text{for} \quad t \geq \tau_i \tag{88}$$

with

$$k(T_c) = \frac{64\pi}{15} g(\theta) J(T_c) \eta(T_c)^3 c_L \alpha_L^{\frac{3}{2}}$$ \quad \text{for} \quad t \geq \tau_i \tag{89}$$

where $T_c$ is the number-average crystallization temperature, $\phi$ is gas-free volume fraction of ice within the pores defined by $\phi = \frac{S_i}{(S_i + S_L)}$, $\alpha_L$ is liquid thermal diffusivity, $\eta(T)$ is a dimensionless temperature-dependent growth parameter (see Equation 9 of Dursch et al.\textsuperscript{228}), $\theta$ is the contact angle of the ice/water/substrate triple line, $g(\theta) = (2 + \cos \theta)(1 - \cos \theta)^2 / 4$ for heterogeneous nucleus growth on a flat surface and $J(T)$ is the pseudo-steady-state nucleation rate (nuclei m$^{-3}$ s$^{-1}$) that has a form based on classical-nucleation theory of\textsuperscript{193,198}

$$J(T) = A \exp \left(\frac{-B}{T(T_{ref} - T)^2}\right)$$ \quad \text{for} \quad t \geq \tau_i \tag{90}$$
where $A$ and $B$ are determined empirically and $T_{ref}$ is the reference temperature (273 K). The values of $J(T)$ can be measured by repeated experimental freezing studies.\textsuperscript{210,228,229} Finally, to obtain $\bar{T}_f(T)$, the definition suggested by Kaschiev\textsuperscript{230} is adopted

$$\bar{T}_f(T) = \frac{1}{J(T)V_o} \quad (91)$$

where $V_o$ is liquid volume of a water-saturated PEFC porous medium.

To validate ice-crystallization kinetics within PEFCs, Dursch \textit{et al.} compared measured to predicted MEA cell-failure time in a simplified isothermal kinetic cold-start model.\textsuperscript{210} Figure 17 compares predicted to measured (symbols) $t_{fail}$ versus subcooling, $\Delta T = T_{ref} - T$, defined as the magnitude of the difference in the temperature of freezing and 273 K. Solid and dashed lines correspond to the kinetic-based approach (equation 87)\textsuperscript{210} and thermodynamic-based approach,\textsuperscript{218} respectively. As subcooling extends beyond $\Delta T = 11$ K, ice-crystallization kinetics is well approximated by the thermodynamic-based approach, since ice crystallization occurs rapidly. However, in the kinetic approach, the particular $\Delta T$ that establishes the “nucleation-limited” regime relies heavily on all heat-transfer and kinetic parameters. Accordingly, these controlling parameters can be adjusted to significantly delay or even prevent ice formation. The impact of the nucleation-limited regime is also shown to agree better with experimental isothermal cold-start data, and the other experimental findings mentioned above.
Figure 17. MEA-cell-failure time, $t_{fail}$, for isothermal galvanostatic start-up as a function of subcooling at a current density of 20 mA/cm$^2$. (Figure reproduced from reference 210).

In addition to freeze kinetics, Dursch et al.\textsuperscript{231} also examined non-isothermal melting in the GDL. It was shown that ice-melting times decrease nonlinearly with increasing heating rate, although the melting temperatures remain at the thermodynamic-based values (consistent with Gibbs-Thomson, equation 86) but are rate limited by heat transfer. Using a moving-boundary Stefan problem, they derived an expression for the melting time,

$$t_{melt} = \left( \frac{\varepsilon_0 \rho L S L \Delta \hat{H}_f}{\beta} \right)^{1/2} \left[ \frac{L^2}{k_{eff}^2} + \frac{2L}{U} \right]^{1/2}$$ \hspace{1cm} (92)

where $L$ denotes bulk-ice or GDL thickness, $\rho_L$ is liquid mass density, $\Delta \hat{H}_f$ is latent heat of fusion per unit mass of ice (taken as positive), $U$ is the overall heat-transfer coefficient, $\beta$ is the heating rate (K/min), and the other variables are as already defined. This melting time should be incorporated into cold-start simulations.
**Catalyst-layer modeling**

In catalyst layers, all of the species and phases discussed above exist in addition to source terms for heat, reactant consumption, product generation, etc. Thus, the conservation and transport equations delineated in above sections are used for modeling, as well as the multiphase flow treatments discussed above. Figure 18 shows a schematic representation of oxygen transport into and through the cathode catalyst layer. Here, one can see that multiple transport mechanisms occur. Due to the pore size, transport through the catalyst-layer thickness is dominated by Knudsen diffusion (equations 17 and 18). At the local length scale, transport through an ionomer and/or water films can become limiting, especially for low catalyst loadings. In terms of the agglomerate, while it is known that primary particles agglomerate during formation, the resultant secondary particles are not necessarily large enough to have significant transport resistances. In this fashion, agglomerates may not exist but then the agglomerate model results in a standard porous electrode one. In many ways, the agglomerate model approach provides an additional parameter to modulate the transfer current density, and although maybe not entirely physical, it does allow insight and better model predictions; it is the standard approach taken for continuum modeling.
Depending on catalyst-layer structure, two different modeling approaches can be employed: homogeneous model\textsuperscript{112,232,233} and agglomerate model.\textsuperscript{110,111,234,235} The homogeneous model applies to an electrode with homogeneous properties and structure in which primary Pt/C catalyst particles are coated with an ionomer thin-film and the effectiveness of electrochemically active Pt surface is 100%. This is typically not the case for standard catalyst layers. The best approach for modeling a catalyst layer would be direct simulations through the actual structure. The reason is that the structures themselves are very heterogeneous and complex, and thus a macrohomogeneous approach may not capture many of the phenomena and interactions occurring. Similarly, unlike the DM, the microstructure complexity demands more complex models than a pore-network-model approach. However, due to the extremely
small pore sizes and the same very complex structures, both virtual reconstructions (as described in a section above) and actual images are not readily available, with only a few studies on the structures.\textsuperscript{236-238} In fact, there is still debate as to how the ionomer is distributed in the cell as well as particle sizes, etc.\textsuperscript{239-242} For example, although often over-looked,\textsuperscript{243-250} it has been known that Nafion ionomer does not penetrate the smaller (primary) pores ($O(20 \text{ nm})$) within the agglomerates in PEFC catalyst layers. This is quite reasonable since the primary structures of Nafion are 30 nm long single chains of ionomer with a diameter of 3 to 5 nm,\textsuperscript{251} or larger aggregates.\textsuperscript{252,253} Thus, even though Nafion is known to adsorb well on carbon-black surfaces,\textsuperscript{237} as also discussed for thin-films below, the Nafion aggregates cannot easily enter the smaller primary pores of Pt/C agglomerates. Thus, the internal areas for reaction may indeed by filled with water or perhaps only by vapor. On top of these complexities is the fact that the structure is not stagnant and evolves during cell operation. For these reasons, macroscopic approaches are still used. Characterization of the catalyst-layer structure and related variables under operating conditions remains a critical need for future model development and validation.

In the subsections below, we examine the critical issues of catalyst-layer modeling across multiple length scales, but with an emphasis on the continuum approaches. First, the treatment of macroscopic modeling of transport in the catalyst layer is presented. Next, the issues that arise with ionomer thin-films within the catalyst layer are discussed, followed by examining mesoscale modeling of transport that necessitates use of double-layer phenomena. Finally, catalyst-layer degradation and its association with transport modeling are discussed.

**Microscale simulations and reconstruction**

Similar to the reconstruction and microscale simulations of DM described in the multiphase flow section, one can also do similar analysis for the catalyst layer.\textsuperscript{254-263} However, since it is a much more
complicated structure with much smaller domain sizes and a lower amount of knowledge and imaging, such simulations and stochastic reconstructions are challenging, with limited image-based approaches being used.\textsuperscript{262} The challenge is offset by the need since catalyst layers are the most important layers. As discussed in the section related to DM, the numerical techniques used for analysis tend to be independent of the method used for reconstruction. Monte Carlo, finite element, finite volume, and lattice-Boltzmann techniques have been commonly applied, but there is still a need for greater understanding of the interaction between materials, particularly the interfacial behavior. For example, as discussed below, ionomer thin-films seemingly are important, but their incorporation and modeling in microscale models is in its infancy. At this stage it is not clear if this type of behavior requires a different set of physics in order to describe it nor if the current numerical methods imposed to undertake analysis on the digital reconstructions are sufficient to accommodate those changes.

Early reconstructions focused on large particles\textsuperscript{255} or lattice techniques.\textsuperscript{249,251,254,255} One of the key questions between the two approaches is whether the resolution of the particles and overall domain is sufficient to determine accurate estimates of the transport properties for the media. The lattice-based approaches typically have relative domain sizes that are larger as different control volumes can be simply specified to belong to a specific phase (ionomer, carbon, platinum, or pore), where as in the particle approach there will exist many control volumes within a particle such that transport within the particle or across the interface are well resolved. It would seem reasonable to assert that the success or failure of either of the two approaches will be a large function of whether the size of the total domain or the interfacial-area aspects dominates the transport phenomena of interest.

More recent efforts try to closer recreate the actual manufacturing process. For example, it is possible to apply rule-sets (similar to that done for the GDL), where one can start with a carbon particle distribution based on the criteria of constrained overlap as shown in Figure 19a. After the placement of
the carbon, the platinum is deposited by randomly seeding a location on the carbon surface similar to a functionalized site that would available in the real process for platinum deposition. This functionalized surface is then subjected to simulated particle deposition and growth that is intended to simulate platinum plating, shown in Figure 19b. Finally the resulting carbon/platinum structure is intruded with an ionomeric phase resulting in the digitally reconstructed morphology, Figure 19c. Simulations can then be carried out on these microstructures to determined effective properties.

Figure 19: Chaining-rule-set stochastic simulations to reconstruct catalyst layers starting with (a) carbon particles, the (b) seeding/plating deposition of Pt, and finally (c) ionomer intrusion based on a sticking probability algorithm.
The aspects of the reconstruction methods underlies a specific reality related to the catalyst layer, which is that to-date key behaviors related to how the microstructures are formed is not fully understood. The development of catalyst layer formation models are a key area for on-going research with only a few selected publications in the area. Other representation on the molecular-dynamics scale have been attempted to further provide characterization of the individual constituents that make-up the layer itself. In particular though, questions about the phase-segregation that occurs with ionomeric materials, the associated processing methods, additives, and surface-interactions between the materials within ink solutions are of interest. It is also interesting to consider that this aspect of manufacturing and the associated uncertain around the physics of formation that lead to one of the primary differences in considering the current methods of stochastic and image-based reconstruction, in that the stochastic reconstructions are meant to encompass the range of variability seen from this formation process.

**Transport within catalyst layers**

The key for modeling catalyst layers is developing the correct expressions for the transfer current between the electron- and proton-conducting phases that represents the reaction as a function of local conditions. As noted, the methodologies for effective properties and multiphase and energy transport across the layer are handled in the same manner as with the DM, albeit with more complicated expressions due to the more complex and heterogeneous structures. For the ionomer thin-films in the catalyst layer, the same approaches as for the bulk membrane can be used (see equations 37 and 38), although their validity is unknown due to the lack of underlying knowledge of the structure/function relationships of ionomer thin-films in the catalyst layers.
The kinetic expressions, equations 57 and 59 for the HOR and ORR, respectively, result in a transfer current between the electronic-conducting (1) and ionic-conducting (2) phases (see equation 11), which is related to the current density in the two phases through equation 23 (neglecting double-layer charging)

\[ \nabla \cdot i_a = -\nabla \cdot i = a_{1,2} i_{b,1-2} \]  

(93)

Assuming the ORR is the only reaction that is occurring in the cathode (i.e., crossover and degradation reactions are ignored), the conservation equation (11) for oxygen in the cathode can be written as

\[ \nabla \cdot \mathbf{N}_{O_2,G} = -\frac{1}{4F} a_{1,2} i_{o,ORR} \left(1 - \Theta_{PO} \right) \left( \frac{P_{O_2}}{P_{O_2}^{ct}} \right)^{m_0} \exp \left( -\frac{\alpha_i F}{RT} \left( n_{ORR} \right) \right) \exp \left( -\frac{\Theta_{PO}}{RT} \right) = \frac{1}{4F} \nabla \cdot i \] 

(94)

The interfacial area of the catalyst with respect to electrolyte and gaseous reactants, \( a_{1,2} \), is often determined by

\[ a_{1,2} = \frac{m_{Pt} A_{Pt}}{L} \]  

(95)

where \( L \) is the thickness of the catalyst layer and \( m_{Pt} \) and \( A_{Pt} \) are the catalyst loading and surface area (which is typically derived from experiment). If there is liquid water in the catalyst layer, this is expected to block the reaction sites. To account for this, one can use an approximation to scale the reaction area

\[ a_{1,2} = a_{1,2}^0 \left(1 - S \right) \]  

(96)

where \( a_{1,2}^0 \) is the specific interfacial area with no water blockage, and the saturation can be determined based on the methodologies described in the multiphase flow section. While simple, this approach is not rigorous, especially since water production is local and may impact the mass-transport to the reaction site without filling the volume. Thus, a better approach is to use a film of water that is generated and retards the gas transport. To do this, one can use the treatment described for thin-films in the next
subsection, although the water thickness will vary with operating conditions, especially due to the local heating at the catalyst site. Modeling, clarifying, and describing these complexities is currently a research need.

The pore structure of catalyst can be classified into intra-agglomerate and inter-agglomerate pores where the agglomerate is essentially the mesoscale structures that contain the reaction sites (see Figure 18). The intra-agglomerate pores determine the utilization of Pt catalyst and inter-agglomerate pores facilitate the transport of gases and liquid. The distribution of these pores determines the balance between the electrochemical activity with mass-transport phenomena. Describing the structure is one of the most complex efforts to be undertaken in transport modeling and is a key future area of integration for different models and modeling scales. Even in the continuum, macroscopic models there are essentially two major length scales that are considered depending on the inter and intra particle/agglomerate interactions. The former or layer length scale is treated using the approaches described in the preceding sections, although with the porosity given by

\[ \varepsilon_{CL} = 1 - \left[ \frac{1}{\rho_{Pt}} + \frac{R_{C/Pt}}{\rho_C} + \frac{R_{C/Pt}R_{I/C}}{\rho_I} \right] \frac{L_{Pt}}{\Delta X_{CL}} \]  

The ionomer volume fraction is written as

\[ \varepsilon_I = \left[ \frac{R_{C/Pt}R_{I/C}}{\rho_I} \right] \frac{L_{Pt}}{\Delta X_{CL}} \]  

These volume fractions are used for the macroscopic transport equations for the gas and liquid phases, where a Bruggeman type expression is used unless there is more detailed knowledge such as mesoscale simulations to determine the properties.  

\[ 245,266 \]
As noted, to model the catalyst layer does not require new equations per se. However, for macroscopic simulations, one would like to use the microscopic phenomena but applied at the macroscopic scale. Such an effort can be achieved by using scaling expressions determined through mesoscale and other modeling approaches, or by modifying the transfer-current source term to account for diffusional losses at the agglomerate or reaction scale. The agglomerate model considers simultaneous reaction and diffusion into an agglomerate where there are numerous primary Pt/C catalyst particles homogeneously mixed with ionomer (see Figure 18). As mentioned, this assumption idealizes the local structure, but does allow for better model predictions; discussions on the impact of the local effects are made in the next sections. Within the agglomerate, isothermal and isopotential conditions are conveniently assumed. The validity of such assumptions were examined based on the magnitude of thermal and ionic transport properties.

The diffusion and reaction into a spherical agglomerate is given by the dimensionless equation (see equation 11)

$$\frac{1}{r^2} \frac{d}{dr} \left( r^2 D_{O_2}^{eff} \frac{dc_{O_2}}{dr} \right) + \frac{1}{4F} a_{1,2\text{ORR},1-2} i = 0$$

(99)

where the effective diffusion coefficient is through the agglomerate. To understand the impact of the agglomerate, an effectiveness factor can be used, which is defined as the ratio of the actual reaction rate to the rate if the entire interior surface is exposed to the conditions outside of the particle

$$E = \frac{4\pi R_{agg}^2 \left( -D_{O_2}^{eff} \frac{dc_{O_2}}{dr} \bigg|_{r=R_{agg}} \right)}{4\frac{\pi R_{agg}^3}{3} \left( -k_{s,m_0} c_{O_2,s}^{m_0} \right)}$$

(100)

where $R_{agg}$ is the agglomerate radius and $k_{s,m_0}$ is the reaction rate of the ORR at the surface conditions.
\[
k_{x,m_0} = \frac{a_{1,2}k_{\text{ORR}}}{4Fc_\text{O}_2^{\text{ref},m_0}} (1 - \Theta_{\text{PtO}}) \exp\left(\frac{-\alpha_2 F}{RT} (n_{\text{ORR},1-2}) \exp\left(\frac{-\Theta_{\text{PtO}}}{RT}\right)\right)
\]

(101)

Thus, one can write the transfer current as

\[\nabla \cdot \mathbf{i}_2 = a_{1,2}j_{h,1-2}E\]

(102)

For a first-order reaction \((m_0 = 1)\), the simultaneous diffusion and reaction into a spherical agglomerate (equations 15 and 11) can be solved analytically to yield an effectiveness factor expression of

\[
E = \frac{1}{\phi^2} (\phi \coth(\phi) - 1)
\]

(103)

where \(\phi\) is the dimensionless Thiele modulus

\[
\phi = R_{\text{agg}} \sqrt[2]{\frac{k_{x,m_0} c_\text{O}_2^{m_0-1}}{D_{\text{O}_2}}}
\]

(104)

which is a measure of the reaction rate to the diffusion rate. Similar expressions hold for other with minimal error.\(^{111,268}\)

As noted in Figure 18, there are possible transport limitations external to the agglomerate due to water or ionomer films. This issue is a critical one moving forward and is discussed in more detail in the next section in terms of its existence, impact, modeling methodology, and underlying genesis.

**Ionomer thin-films**

The oxygen-transport resistance in a cathode has been measured by limiting-current measurements using diluted oxygen.\(^{234,269}\) It was found that the resistance increased as Pt loading decreased.\(^{110-112,233,234,270}\) The voltage-losses scale with the Pt surface-area specific current density (as opposed to
geometric-area specific current density) after correction for bulk (channels, DM, catalyst-layer length scale) transport losses by suitable models. The local resistance is thus assumed to exist at or near the Pt surface to account for the unexplained higher voltage loss associated with lower Pt loading. As shown in Figure 18, the reactive sites (whether bare or in an agglomerate) are expected to be at least partially covered by thin-films of ionomer in order to provide proton conduction to the active site. In this confined structure, the properties of are expected to differ significantly from bulk membranes, meaning that one cannot treat the ionomer film in the catalyst layer as the same polymer used as PEM.

Figure 20. Water-uptake isotherms for Nafion in various PEFC catalyst layers as a function of relative humidity compared with that of bulk Nafion membrane.

It is known that the properties related to the transport functionalities of the ionomer in a catalyst layers differs from that of the bulk electrolyte. As shown in Figure 20, water-uptake isotherms for Nafion in the catalyst layers show much lower water contents compared to bulk Nafion membrane. Also, water uptake in catalyst layers has been shown to increase with increasing loading of platinum, which
improves wetting properties and hydrophilicity,\textsuperscript{131,271} and with increasing ionomer content,\textsuperscript{271,272} although in general the catalyst layer’s ionomer water uptake has been consistently shown to be depressed.\textsuperscript{131,272} With the lower water contents (and perhaps the existence of different morphology and confinement driving the lower water uptake), it is not surprising that the ion conductivity is likewise depressed,\textsuperscript{272-274} and is a function of ionomer content.\textsuperscript{271,272,275,276} However, it should be noted that some studies suggest similar proton conductivity in catalyst layers as in bulk,\textsuperscript{277,278} which may be due to measurement methods or perhaps anisotropic conductivity; more data is required. Finally, it is important to realize that understanding ionomer thin-film behavior is key not only for PEFC catalyst layers, but also for the characterization of bulk membrane surfaces, where the near-interface regions could be interpreted as thin-films.

\textit{Ionomer thin-film modeling}

As discussed above, the processes to model in the cathode catalyst layer include proton conduction and oxygen transport to the Pt surface where they react together with electrons to form water, which has to be removed from the electrode. In this section, we specifically examine modeling transport through the ionomer thin-film and its associated effects. Furthermore, as modeling liquid-water transport involves much complexity with a large amount of uncertainty (more data is required), the discussion below neglects liquid water. As the proton-transport resistance is needed to model proton conduction through the ionomer thin-film, this can be readily measured by AC-Impedance as a function of relative humidity and temperature,\textsuperscript{279} or derived from property relationships and is not discussed below.

When adding the film, the governing equation for oxygen transport (see equation 99) becomes
where \( z = r / (r_{agg} + \delta_{film}) \) and \( \delta_{film} \) is the film thickness. One can then use the same agglomerate approach as described above. For accounting for transport into and through the film, a two-stage process is used, comprised of transport through the film

\[
N_{O_2} = -D_{film} \nabla c_{O_2} = -\frac{D_{film}}{RT} \nabla p_{O_2}
\]  

(106)

and the slow oxygen dissolution process at the gas/ionomer interface,\(^{234,280,281}\)

\[
N_{O_2} = -a_{film} k_{O_2,film} (c_{O_2,eq} - c_{O_2,i/g}) = -\frac{a_{film} k_{O_2,film}}{RT} \left( H p_{O_2} - p_{O_2,i/g} \right)
\]  

(107)

where \( a_{film} \) is the specific surface-area of the film, \( k_{O_2,film} \) is the rate constant for oxygen dissolution, \( H \) is Henry’s constant, and \( p_{O_2,i/g} \) is the equivalent pressure within the ionomer at the interface with the gas. Combining the above expression with that of equation 106 and assuming a linear gradient in the very thin film, results in

\[
N_{O_2} = \frac{1}{RT} \frac{H p_{O_2,ext} - p_{O_2,surf}}{R_{O_2,film}}
\]  

(108)

where \( R_{O_2,film} \) is the transport resistance of oxygen through ionomer film,

\[
R_{O_2,film} = \frac{\delta_{film}}{D_{film}} + \frac{1}{a_{film} k_{O_2,film}}
\]  

(109)

Alternatively, if one does not independently know the solubility and diffusivity, then the permeation coefficient \( (\psi_{O_2} = DH) \) can be used. In addition, since the film thickness and its transport properties are typically both unknown, one can just use the resistance for modeling.
At steady state, the flux given by equation 108 is equal to the flux due to reaction and diffusion in the agglomerate; therefore, the unknown concentrations can be replaced. Using the resultant expression in the conservation equation (23) yields

\[
\nabla \cdot \mathbf{i}_1 = 4Fp_{O_2,\text{ext}} \left( \frac{1}{R_{O_2,\text{film}}} + \frac{1}{k_s,m_0 E} \right)
\]

and a similar one can be derived for the HOR.

The above expression is the governing equation for the transfer current density and includes both the film and agglomerate resistances. Upon inspection, one can see that as the current density increases (i.e., \(k_{s,m_0}\) increases), the film resistance becomes more significant and limiting. Thus, the impact of the ionomer thin-film on cell performance is expected to be more pronounced at low Pt loading due to the higher local oxygen flux to each reaction site.

As mentioned, the local resistances to the Pt surface increase with decreasing Pt loading. For example, Nonoyama et al. measured the local resistances of ~0.04 s/cm at 0.20 mg/cm\(^2\) and 0.06 s/cm at 0.10 mg/cm\(^2\), where the other transport resistances have been removed through systematic variations in gas composition and pressure. This increase could be due to decreased ionomer surface area for effective oxygen permeation and subsequent higher fluxes through the ionomer thin-film to reach the catalyst site. Alternatively, ex-situ measurements have been made to determine the local resistance. The diffusion-limited current density in a planar Pt electrode with spin-coated ionomer thin-films was measured as a function of film thickness, and the local resistance was obtained by extrapolating the reciprocal limiting current density to zero film thickness. With the assumption that the resistances result from slow oxygen dissolution from the gas phase into the ionomer, the local
resistance was incorporated into the agglomerate model with the agglomerate size being a fitting parameter to match the measured cell performance. The resulting agglomerate size is close to the primary Pt/C catalyst particle size, consistent with microscopy observation of the catalyst layer, which did not reveal any large agglomerates.\textsuperscript{280}

The local resistance has been measured \textit{in-situ} and \textit{ex-situ} for use in predicting the performance of the low Pt-loaded electrodes. However, its origin is still unclear. Hypothetically, the local resistance can be attributed to slow oxygen dissolution process at the gas/ionomer interface,\textsuperscript{234,280} and/or strong interaction between the Pt surface and the sulfonate groups in ionomer leading to either a decrease in the effective Pt surface area, or ionomer structural changes in the thin-films near the Pt surface that lowers oxygen permeability as discussed below.\textsuperscript{282} Thus, the measured local resistance is a sum of three components from gas phase to the Pt surface: the interfacial resistance at the gas/ionomer interface, the bulk resistance of the ionomer film, and the interfacial resistances at ionomer/Pt interfaces. Quantification of each resistance would provide fundamental knobs to mitigate the voltage losses due to the local resistance via a combined modeling and experimental approach.

The relative importance of the two interfacial resistances was investigated by modeling the performance and limiting-current measurements on cathodes having the same thickness, ionomer content, and macroscopic structure but different Pt loading and Pt.\textsuperscript{283} The Pt nanoparticle distribution was varied by diluting various wt-% Pt/C catalysts with bare carbon support. The experimental data show that at a given Pt loading, voltage loss increases with Pt particle density and the electrode resistance trends accordingly as shown in Figure 21. The model-data comparisons demonstrate that the two interfacial resistances are equally important, particularly for the low Pt-loaded electrodes made of high wt% Pt/C catalyst.\textsuperscript{283} As shown in Figure 21b, the incorporation of the interfacial resistance is key for matching the data. This analysis stresses the need for incorporating the distributed resistances at the
thin film (e.g., equation 109) in more detailed models for performance prediction and electrode design. Both models and especially novel experiments are critical areas for quantifying and understanding thin-films and their impacts on PEFC performance.

Figure 21. Pressure-independent oxygen transport resistance from single-particle model and experiments as a function of ionomer film to Pt surface area ration for (a) varying Pt loading with most dispersed samples (no bare carbon dilution) and (b) constant Pt loading with varying Pt dispersions. Reproduced from reference 283.
**Ionomer thin-film structure**

In the previous sections, we examined the possible importance of ionomer thin-films in impacting the reactant mass transport to the electrocatalyst site and the overall electrochemical response of the system. One expects that as the ionomer thickness decreases from a membrane to a thin-film, associated transport properties could exhibit strong deviations from their bulk-polymer values. For example, one can envision that the ionomer thin-film may behave entirely like the interface in a PEM.\(^{272}\) It is well known that the behavior and properties of the ionomer are controlled by a balance between the chemical (or solvation) and mechanical (or deformation) energies,\(^{62,90,284}\) and thus exploring the structure of ionomer films on various substrates is a worthwhile endeavor. Furthermore, studies on thin-film behavior are keys not only for PEFC catalyst layers, but also for the characterization of the bulk-membrane surface, where the near-interface regions could be interpreted as thin-films.

When a polymer is confined to thicknesses comparable to its characteristic domain size, its properties and morphology differ from the analogous bulk materials. Thin-film confinement and substrate/film interactions are known to affect the phase behavior of uncharged block-copolymer systems.\(^{285-289}\) Even though confinement effects in polymer films have long been of interest, films of phase-separated ionomers, such as PFSAs, have gained attention only recently.\(^{290-299}\) Ion-containing polymers are more complex than many widely studied polymers due to the presence of solvent/ion electrostatic interactions, hydrogen bonding, and less defined chain structure, and their self-assembly is expected to be affected by wetting interactions at both the substrate and free (vapor\-liquid) interfaces as well as by topological confinement effects. Thanks to recent studies on Nafion thin-films reporting confinement-driven changes in properties, it has been acknowledged that the current understanding is
still far from complete due to the significant material-parameter space for thin-films compared to the bulk (e.g., film thickness, casting and processing conditions, substrate, etc.).

Interactions of the ionomer with the catalyst-layer interface as a membrane and within the catalyst layer as a thin-film result in highly complex material behavior that is highly dependent on the environmental conditions, ionomer thickness, and the material interactions with carbon and platinum. Most studies investigate Nafion thin-films on a silicon substrate, a commonly used model substrate in thin-film applications. Many of these studies reported a decrease in water content or swelling,291,300,301 ionic conductivity,292,294,302 and/or mobility and diffusion,290,291,293 for the Nafion thin-films as shown in Figure 22. The exact magnitude of these changes largely depends on the processing conditions, thickness, and the substrate. Also, often swelling is used for measuring uptake behavior, although one does not yet understand the dimensionality of swelling in the confined films.303 In general, for Nafion thin-films, a significant deviation from bulk behavior is observed in uptake and related transport properties when the films are confined to thicknesses of less than 100 nm. Typically, there is a decrease in water content until one gets to very thin-films (< 20 nm) where the lack of being able to form crystallites results in an increase of water content.292,303 The values and trends are in agreement with those derived from catalyst-layer studies (see Figure 20) in terms of both water contents as well as time constants for water uptake.131,272,292 This agreement suggests that ionomer thin-films can serve as model systems to study and elucidate the transport phenomena and related structure/function relationships in catalyst-layer ionomers. The conductivity of Nafion thin-films also decreases by an order-of-magnitude.294,302 An increase in activation energy for conductivity was reported for thinner films which suggest intrinsic changes in morphology controlling the conduction mechanisms.294,302,304
Figure 22. Water uptake and conductivity for ionomer thin films. (Data take from references 292 and 303.)

Understanding the origins of these confinement-driven changes in transport properties is possible only by exploring the surface structure and internal morphology of the thin-films. Recent studies have demonstrated that confinement effects and wetting interactions result in different surface and near-edge morphological properties of thin-films, as shown by Grazing-incidence small-angle X-ray Scattering (GISAXS),292,296,300,301,303 reflectivity,300,305 AFM,292,296,304,306,307 TEM,292 fluorescence,291 and positron annihilation.306 Although surface characterization techniques (such as AFM) are commonly used on thin-films, conductive AFM is yet to be implemented for thin films to explore their transport behavior. GISAXS has been shown to be extremely useful to investigate the orientation of the domains at and below the thin-film interfaces. Thus far, GISAXS has been employed for dry and hydrated films with thicknesses of 5 to 250 nm.292,296,301 GISAXS analysis indicates that the origins of the thickness and substrate effect are related to the alignment and reorganization of the water domains in the polymer matrix, which are also controlled by the interactions between the substrate and polymer film. On silicon, Nafion films of less than 25 to 50 nm thickness were observed to exhibit negligible scattering without a strong ionomer peak in GISAXS experiments, while showing more mixing of smaller domains as confirmed by TEM,292 implying a change in phase-separation behavior. Another evidence of change in
the thin-film’s surface morphology is the thickness-dependent surface wettability: thicker films have a hydrophobic surface whereas sub-50 nm thick films exhibit a hydrophilic surface.\textsuperscript{308}

More recently, GISAXS of Nafion and other PFSA thin-films have been investigated on other substrates including carbon and metals.\textsuperscript{303} Also, a recent neutron reflectivity study on Nafion thin-films suggested that domain orientation closer to the support could be parallel to the substrate or random on silicon and gold, respectively.\textsuperscript{305} Furthermore, in a simulation study, Nafion’s backbone chains were shown to be attached to graphite substrate surface, suggesting a hydrophilic external surface.\textsuperscript{241} It was also argued that hydration of thin-film without swelling increases ion mobility.\textsuperscript{241} Thus, there is still significant knowledge gap when it comes to the morphology and behavior of thin-films interacting with Pt and carbon in environments that are relevant to PEFC catalyst layers. The ionomer/substrate interface must be studied in more detail, both experimentally and computationally, to unveil the nature of chemical interactions between the carbon and Pt substrates and polymer backbone, side-chains and acid groups, as well as their influence on transport of specifies, especially in the presence of water and other solvents. Such studies will not only reveal how the thin-films may affect transport, but also perhaps the poisoning effect of the ionomer on the catalyst.\textsuperscript{309,310}

Once understood, ionomer/substrate interactions at nanoscales can be exploited to model structure/property relationship of catalyst layers at mesoscales. For instance, using dissipative particle dynamics, conductivity in CL ionomer on a carbon support was shown to be anisotropic due to the layered structure in the electrolyte.\textsuperscript{311} Since orientation of water domains in thin films are already observed to change with wetting interactions and substrate (using GISAXS), advances in experimental and simulation techniques must be taken advantage of to obtain more information on size-distribution and orientation of domains. Only by understanding the role of film thickness and substrate/film interactions in structure/functionality of thin-film ionomers, can one develop tools to achieve optimum
ionomer content, better Pt utilization or reduced Pt loading, and ultimately improved catalyst-layer performance. For the latter, there is still a critical need to explore the transport properties of thin-films including, but not limited to, oxygen transport, ionic conductivity, diffusivity, thermal properties, and mechanical properties, both in the plane and thickness directions. It is also important to carefully monitor the timescales during any experimental study to investigate the dynamics response and equilibrium behavior of thin films.

**Mesoscale electrode analysis and ionomer-free electrodes**

In this section, we discuss the analysis of mesoscale transport in catalyst layers. The prior sections focused on reconstructed catalyst-layer structure, multiscale modeling, and upscaling to macroscale transport properties. Such approaches often assume idealized Nafion/Pt interfaces. Below, the nonideal features of the electrode structure and the associated transport resistances are discussed. In particular, the impact and modeling of incomplete ionomer coverage on Pt/C catalyst as well as the physics of proton transport on next-generation ionomer-free electrodes, such as 3M’s nanostructured thin-film (NSTF) electrodes, are introduced.

An important class of alternative electrode architectures is ultra-thin ionomer-free electrodes, exemplified by 3M’s NSTF electrodes. These promising electrodes have thicknesses from 0.2 to 1 µm and are ionomer-free polycrystalline Pt whiskers that are partially embedded in the membrane upon cell assembly. They have demonstrated good mass activity and durability, but have been shown to be sensitive to flooding and liquid water, especially at lower temperatures, due to their thinness. It is desirable to pursue such structures, but questions remain regarding possible proton limitations and the ion-conduction mechanism.
Although experimental work on ion transport in ionomer-free regions is sparse, theoretical modeling of ion transport in these ionomer-free regions is even less developed. In state-of-the-art PEFC electrode models, it is often assumed that the primary pore structure is filled with Nafion\textsuperscript{249} or water\textsuperscript{315} and that the ionic transport within primary agglomerates is infinitely fast and any ohmic losses from within the agglomerate’s primary pores are neglected.\textsuperscript{243-250} Yoon and Weber\textsuperscript{111} have shown that this is a reasonable assumption if the proton conductivity within the agglomerate is > 0.001 S/cm. A conductivity in this range is consistent with a high volume fraction of Nafion and bulk-like Nafion conductivity. However, the effect of inter-agglomerate resistances could be significant in ionomer-free pores and in dry conditions.

\textit{Ion transport in ionomer-free zones}

Nafion in PEFC electrodes acts as a proton conductor and its fixed high proton concentrations ensure that the ORR is not ion-transport limited. In ionomer-free zones of conventional PEFC electrodes and NSTF electrodes there is no Nafion to aid in proton transport. Ion conductivity of bulk neutral water is about seven orders of magnitude lower than that of Nafion. The question remains open regarding possible proton transport mechanisms present in the water-filled regions of PEFC electrodes. Over the past two decades two major transport mechanisms have been proposed to explain ion transport in ionomer-free regions: 1) adsorbed H-adatom diffusion along the electrode surface,\textsuperscript{316-319} and 2) water-mediated conduction of protons.\textsuperscript{320,321}

Several studies have reported or investigated the electrochemical activity of ionomer-free electrode regions beyond the interface with the PEM.\textsuperscript{316-318,322,323} Beginning with the work of McBreen,\textsuperscript{316} the majority of studies have focused on evaluating ECSA with underpotential deposited (UPD) hydrogen in cyclic-voltammetry experiments.\textsuperscript{316-319} These experiments demonstrated that the entirety of the
continuous Pt surface connected to the PEM was accessible to UPD hydrogen. However, it is apparent from those results (particularly from the work of Paulus et al. using model electrodes)\textsuperscript{317} that the electrode area is accessible because of Faradaic reactions at the PEM interface followed by surface diffusion of the adsorbed hydrogen ad-atoms along the Pt/water interface (second proposed mechanism). Thus, although the Pt surface area is accessible by CV measurement, it is not necessarily electrochemically active. In terms of RH, several groups have reported up to a 50% drop in ECSA when the RH is reduced.\textsuperscript{324,325} Electrochemical-impedance-spectroscopy measurements have demonstrated that the conductivity of the water in electrodes without ionomer is many orders of magnitude higher than that of bulk water and approaches that of the Nafion in Nafion-bound electrodes.\textsuperscript{322,323,326}

Recently, Litster and coworkers\textsuperscript{277,327} have performed directed measurements of electrode conductivity during PEFC operation using microstructured electrode scaffold (MES) diagnostics. They observed conductivities that were 3 to 4 times higher than expected from the bulk conductivity of Nafion and its estimate volume fraction and tortuosity in the electrode. This finding agrees with prior transmission-line analysis of N$_2$/H$_2$ impedance measurements,\textsuperscript{328} but contradicts \textit{ex-situ} measurements observing reduced conductivity of Nafion thin-films as discussed in the previous section, but these films may be anisotropic.\textsuperscript{292,294} It was suggested that the increased conductivity was a result of surface-supported transport on the high-surface-area catalyst and proton transport through water films (second proposed mechanism). Again, there was a strong dependent on RH with significantly lower conductivity at RH < 60%. Finally, one should also note that during operation, there are often either contaminant ions or membrane-degradation products, which can alter the charge carrier concentration and ionic strength of the water.

There are only a few modeling studies addressing proton transport in Nafion-free electrode domains.\textsuperscript{321,329-331} The Poisson-Nernst-Planck equations can still be applied to describe ionic
concentrations and potential distributions within water-filled domains. These continuous equations do not accurately describe the physics in the vicinity of the water/electrode interface because they assume ions to be point charges and the solution to have a constant dielectric permittivity. Under high applied electric fields or when ion concentration in solution is high, the Poisson-Nernst-Planck theory does not handle well the ion crowding effect at the interface. Essentially, it states that the electrode can be charged to an infinite capacity. There is also a finite physical limit to how closely solvated ions can approach the electrode. When electric potential is applied to the electrode, there is a preferential orientation of solution dipoles induced by the electric polarization. In Poisson’s equation, the dielectric permittivity has to be reduced to account for this interfacial polarization. Given these limitations to the applicability of the continuous Poisson-Nernst-Planck equations and constrained, mesoscale geometries of the electrode domains, there is a need of a model that can address these challenges. For decades, EDL theories have been effectively complementing Poisson-Nernst-Planck theory, bridging these continuum equations with discrete physics at the solution/electrode interfaces.

*Electric-double-layer model for a PEFC electrode*

Whenever there is an interface separating solid and liquid, the mobile species such as ions, electrons, and water dipoles will orient in a way to minimize the free energy of these two phases. In general, EDLs form at solution interfaces due to surface dissociation/association reactions, specific adsorption, applied potentials, and ionomer layers. Figure 23 shows the general EDL structure for a positively charged electrode (e.g., a PEFC cathode). The positive surface charge is due to a depletion of electrons caused by electrochemical polarization of the electrode. Next to the solid surface are three main double-layer regions: (1) the compact, Stern layer, (2) the diffuse layer, and (3) the bulk, electrically neutral solution. This conceptual model is termed the Gouy-Chapman-Stern-Grahame (GCSG) model. The GCSG
model is the most comprehensive form among the EDL models showing good agreement with experimental capacitance measurements. In this microscopic model, a local electroneutrality exists between the charge in solution, \( q_s \), and that on the metal surface, \( q_m \),

\[
q_m = -q_s = -(q_H + q_d)
\]  

(111)

where \( q_d \) is the charge in diffuse layer, which arises from competing migration and diffusion processes, and \( q_H \) is the charge within the Stern layer.\textsuperscript{332} The plane between the Stern and diffuse layers is termed the outer Helmholtz plane (OHP) – the plane of closest approach for fully-solvated ions. In this case, partially solvated ions are located within the Stern layer with their centers along the inner Helmholtz plane (IHP). Although hydrodynamically immobile, ions in the Stern layer are known to have mobilities on the same order as the bulk and can be a large contributor to the ionic conductivity in capillaries and porous media.
As Figure 23 shows the GCSG model implies that the potential difference between the electrode, \( \Phi_m \), and bulk liquid, \( \Phi_l \), has two contributions:

\[
\Phi_m - \Phi_l = (\Phi_m - \Phi_H) + (\Phi_H - \Phi_l)
\]  \hspace{1cm} (112)

where \( \Phi_H - \Phi_l \) is the potential drop within the diffuse layer and \( \Phi_m - \Phi_H \) is that within the Stern layer. For interfaces with thin double layers (i.e., PEFC electrode|Nafion interface) most of the potential drop occurs in the Stern layer. For water-filled pores of PEFC electrodes with thick and overlapped EDLs (due to low ionic concentration and confined geometry) the potential drop within the diffuse layer contributes significantly.

Commonly, water-filled pores within porous PEFC electrodes and water films on NSTF electrodes are modeled with simplified cylindrical axisymmetric domains.\(^{321,330}\) Poisson-Nernst-Planck theory
formulated by equations 24 and 28 describes transport of ions and potential distributions within the diffuse layer. For cylindrical pores, analytical solutions exist for equilibrium EDLs where Poisson-Nernst-Planck equations simplify to the Poisson equation with a Boltzmann distribution for ions (see equation 25).333-336 This formulation is restricted to ion transport in radial direction and assumes that the pore centerline potential is constant in the axial direction. The solution to the potential distribution within the diffuse layer is

$$\Phi(r) = -2 \ln \left(1 - br^2 \right)$$

(113)

where $b$ depends on the electrode surface charge or applied potential, $r$ is a radial distance from the center of the pore, and $\Phi(r)$ is a potential within the pore relative to the centerline pore potential.

Water-filled pores are bounded on the outside by Nafion film with high proton concentration balanced by the sulfonic-acid groups. Within Nafion, the potential can be assumed to be $\Phi_0 = 0$ referenced to SHE as the electrochemical condition within the bulk Nafion phase approximates a 1 M acid electrolyte. Due to orders of magnitude difference in ionic concentrations for bulk water and ionomer, a Donnan potential difference describes the potential drop at the water|Nafion electrode. The Donnan potential difference effectively links the centerline water-pore potential and bulk Nafion. It can be approximated with the solution to Poisson-Boltzmann equation applied to a system with proton-penetrable membrane adjacent to water,337,338

$$\Phi_{DON} = \Phi_0 - \Phi_t = \frac{kT}{ue} \arcsin \left( \frac{zN_{N,H^+}}{2un_{H^+}} \right)$$

(114)

where $N_{N,H^+}$ is the density of charged groups within membrane and $n_{H^+}$ is centerline concentration.

To couple a diffuse-layer potential distribution described by equation $\Phi(r) = -2 \ln \left(1 - br^2 \right)$ (113) to the electrode potential or its surface charge, an additional boundary condition is
required. A common approach is to separate interfacial differential capacitance into a series of capacitances. Dividing equation $\Phi_m - \Phi_l = (\Phi_m - \Phi_H) + (\Phi_H - \Phi_l)$ by the metal charge and differentiating it, one obtains

$$\frac{\partial (\Phi_m - \Phi_l)}{\partial q_m} = \frac{\partial (\Phi_m - \Phi_H)}{\partial q_m} + \frac{\partial (\Phi_H - \Phi_l)}{\partial q_m}$$

(115)

$$\frac{1}{C} = \frac{1}{C_H} + \frac{1}{C_d}$$

(116)

where, $C$ is the total interfacial capacity, $C_H$ is the Stern layer capacity, and $C_d$ is the diffuse layer capacity. The next step is to apply Gauss’s law to a control volume of the Stern layer, neglecting specific adsorption in the IHP and assuming metal to be an ideal conductor (electric displacement is zero). Along with integration of Stern-layer capacitance and setting a lower integration bound to the metal’s potential of zero charge (PZC) a Robin boundary condition can be obtained:

$$\frac{\varepsilon_r \varepsilon_0}{C_H} \frac{d\Phi}{dr}_{\text{electrode}} = \Phi_m - \Phi_H - \text{PZC}$$

(117)

where $\varepsilon_r$ is a dielectric permittivity of Stern layer. Depending on the limits of integration, different variations of equation $\frac{\varepsilon_r \varepsilon_0}{C_H} \frac{d\Phi}{dr}_{\text{electrode}} = \Phi_m - \Phi_H - \text{PZC}$ exist in literature. The PZC is an important intrinsic physical property of the electrode that has often been overlooked in a modeling community. Herein, a brief discussion is provided.

When considering EDLs on metal electrodes, the amount of surface charge being balanced by the ions depends upon the difference between the electrode potential and its PZC, or more specifically, the potential of zero free charge. The PZC is the potential for which there is no free charge at the metal interface with the solution (i.e., there is no excess of positive or negative charge in the solution). Thus, if
the ion concentration of the bulk is low (as in the case of pure water), the conductivity is extremely low at the PZC. For electrode potentials above the PZC, there is excess positive charge in the metal. Likewise, for electrode potentials below the PZC, there is excess negative charge in the metal. Considering the Pt-water electrode system, for potentials above the PZC, there is a depletion of protons, and, below the PZC, there are accumulated protons. For reference, experimental studies have determined the PZC of Pt to be between 0.2 and 0.3 V vs SHE (commonly taken to be 0.26 V vs. SHE)\textsuperscript{344,345}.

Overall, the capability of the EDL model applied to a PEFC water-filled electrode includes a prediction of local ion conductivity and radial, as well as axial, potential distributions within the pore approximated with a cylindrical geometry. The EDL model can be extended to include additional physics, such as: electrode’s surface chemistry (e.g. surface-complexation model),\textsuperscript{330} electrode’s electron spillover effect (e.g. space-charge layer),\textsuperscript{330,346} different solution impurities, and specific adsorption at the IHP (modified Stern isotherm). The EDL model assists with resolving mesoscale transport phenomena coupled to reaction kinetics. Under operating conditions in a PEFC, a current is drawn from the cell and the EDL model can predict local electrochemical reaction generation based on the local reactant concentrations and potential distributions. When the model incorporates both radial and axial dimensions, the reactant gas transport is modeled with Fick’s law and the OHP is assumed to be a plane where the electrochemical reaction is occurring, as it is a plane of the closest approach for hydrated ions. Local current density is modeled with Butler-Volmer or Tafel equations with local OHP reactant and product activities. For macroscale transport models the reaction kinetics equations use the electrochemical reaction driving force between the bulk metal and electrolyte (or ionomer) phases. In the EDL models the local reaction overpotential, $\eta_{\text{OHP}}$, is the reaction driving force, which can be related to a macroscopic surface overpotential, $\eta$.}

100
\[ \eta_{OHP} = \eta - (\Phi_{OHP} - \Phi_0) \]  

Essentially, the potential difference across the Stern layer is the reaction driving force in the EDL models. This potential difference adjustment is also termed the Frumkin correction.\(^{321,330,331,341-349}\) The diffuse-layer potential only affects the interfacial electrokinetics by modifying the activities of reactant species at the OHP.

**Pore-level models**

Macroscopic PEFC models that consider the presence of EDLs in water domains are few.\(^{321,329-331}\) As noted above, the electrode models utilize the Poisson-Nernst-Planck equations coupled with the above GCSG theory and a geometry often consisting of a water-filled pore with perhaps ionomer at one end. To couple the Poisson-Nernst-Planck equations with GCSG theory, the Poisson-Nernst-Planck steady-state equations (equations 28 and 24) are applied on ionomer and water domains. The aim is to solve for potential distribution and proton and hydroxide concentrations. These second-order differential equations require two boundary conditions for each variable. The electrode can be modeled as a 1D domain adjacent to the water. In the ionomer there are two types of ions present: negative sulfonic groups that are stationary, and mobile protons counterbalancing them. In the water domain, the transport of hydroxide and protons is modeled. The outer edge of ionomer serves as a reference for concentrations in the Nernst-Planck equation (e.g. setting them to bulk value in Nafion) and potential in Poisson’s equation (0 V vs SHE). The second set of boundary conditions is considered along the water/electrode interface. For the Poisson equation, a Robin boundary condition (equation 118) should be applied, where \( \Phi_m - PZC \) is the applied electric potential relative to the PZC (both are known), and \( \frac{\varepsilon_r \varepsilon_0}{C_H} \) is constant (i.e., assuming Helmholtz capacitance is constant). For Nernst-Planck, insulation for
hydroxide ions is used, whereas a flux of protons due to the reaction consumption (ORR in cathode) or generation (HOR) is specified. The reaction (e.g. ORR in the cathode) along the water/electrode interface can be described with Tafel equation 58), where the overpotential is set to equation 118 and local activities are used.

The simulation results of Chan and Eikerling\textsuperscript{321} provided two important pieces of insight into the effective operation of ionomer-free electrodes. First, they showed that the primary impact of ionomer-free zones is the reduced reaction current density due to the depletion of reactant protons, rather than a reduction in conductivity and a long-range transport limitation. Second, the work highlighted the critical importance of the metal electrode’s PZC. In order for their model to achieve current densities consistent with those measured for the NSTF electrodes, they had to assume PZC values greater than 0.7 V; these values are well above the range of values experimentally measured for Pt electrodes.

Zenyuk and Litster\textsuperscript{330,331} calculated that incomplete ionomer coverage did not significantly influence a Pt/C anode’s performance at PEFC current densities because protons are not a reactant for the HOR. More recently, the model was extended for Pt surface cathodes with ionomer, such as NSTF electrodes.\textsuperscript{331} The model included finite-thickness domains that captured the Stern layer and space-charge layer in the metal so specific adsorption and dipole contributions to the PZC could be included. In addition, due to large pH variations, the model included non-equilibrium water dissociation and hydroxide-ion transport, with the associated alkaline ORR mechanism. The results showed that the pH quickly increases away from the Nafion interface, particularly at high potentials near open circuit (much higher than the PZC). At a distance more than 10 nm away from the Nafion interface, the concentration of OH\(^{-}\) is many orders of magnitude higher than that of H\(^{+}\). Thus, at high potentials, the ORR on Pt is dominated by the alkaline mechanism and the overall, high conductivity is due to the high concentration of OH\(^{-}\). This suggests the high conductivity measured experimentally may not be the conductivity of
protons, but rather hydroxide ions. The results also show that as the potential is reduced, there is less proton exclusion from the pores and the acidic ORR mechanism provides a greater contribution to the overall current density.

While some progress and understanding has been made recently on mesoscale transport of ions and in ionomer-free electrodes, there is still much more work that is required going forward. For example, it is important to develop a modeling framework that can effectively scale-bridge the above single-pore and double-layer models with macroscopic PEFC models. In this fashion, issues such as water sensitivity simultaneous with proton sensitivity of ionomer-free electrodes can be studied, and optimized designs and operating schemes elucidated. There is also a need for further experimental studies into ionomer-free zones and electrodes in terms of identifying the EDL potentials, impact of surface structures, and ion mobilities. For example, ion mobilities at the interface, including within ice-like water structures and liquid/vapor-like interfaces are not well-resolved experimentally or by simulation. The proton mobility could be significantly increased within these water networks and could dramatically influence transport-model predictions.

**Durability and degradation**

The durability of PEFCs remains a major barrier to their commercialization for stationary and transportation power applications. Power transients, shut-down/start-up, temperature cycling, and RH cycling, are all operating conditions imposed by the rigid requirements of various applications. These operational transients induce a number of chemical and structural degradation mechanisms on MEAs that change the transport properties of both the materials and the structures, and consequently their performance. Major degradation mechanisms involve the catalyst layer and are reviewed below.
These mechanisms include corrosion of the catalyst support, electrocatalyst degradation which includes platinum dissolution, migration and reprecipitation and loss of alloying constituents, and ionomer degradation and restructuring. These processes can cause changes in PEFC performance due to loss of catalyst utilization, electronic and protonic conductivity, and reduction in the transport rates of the gases and product water. Below, they are discussed in terms of their general impacts on transport modeling as well as modeling of the related transport processes themselves.

**Carbon corrosion**

A primary cause for changes in the structure of the catalyst layer is the corrosion of carbonaceous electrode supports leading to electrode collapse, changes in porosity, and reduced active catalyst area. The overall corrosion reaction of carbon in PEFCs can be expressed as

\[
C + 2H_2O \rightarrow CO_2 + 4H^+ + 4e^- \quad E^0 = 0.207 \text{ V} \quad (119)
\]

Thus, this reaction is thermodynamically favorable at the potentials at which the cathode normally operates. However, the kinetics for carbon corrosion at normal cathode potentials is usually slow enough to support a decade of service life. The reason for the slow kinetics has to deal with going through high-potential intermediates. Most mechanistic descriptions of carbon corrosion proceed through a carbon oxide or hydroxide intermediate. The following scheme is exemplary

\[
C^* + H_2O \leftrightarrow C-OH + H^+ + e^- \quad E^0 = 0.2 \text{ V}_{\text{RHE}} \quad (120)
\]

where \( C^* \) is a carbon defect site. The C-OH group can be further oxidized to form a C=O group at a potential of \( E^* = 0.8 \text{ V} \) through the reaction

\[
C-OH \leftrightarrow C=O + H^+ + e^- \quad E^0 = 0.8 \text{ V}_{\text{RHE}} \quad (121)
\]

At a higher potential of \( E^* = 0.95 \text{ V} \), the surface C-OH groups are oxidized to form carbon dioxide that exits the system leaving behind a new defect site,
This scheme captures the carbon dioxide emission at higher voltages from the oxidation of the unstable carbon surface.

As noted, kinetics typically are sluggish enough for carbon corrosion that in the typical PEFC potential range (0.0 to 1.0 V), there is not significant oxidation, especially below 0.9 V. A caveat is that carbon oxidation to CO₂ can be measured and is promoted by potential cycling. The long-term relationship between carbon corrosion and load cycling (including open-circuit dwell time) is, as of yet, relatively undetermined. To incorporate and model carbon corrosion, a Tafel expression is used as the reaction is essentially irreversible,

\[
i_{\text{COR}} = i_{\text{COR}}^0 \exp \left( \frac{\alpha_{\text{COR}} F}{RT} \left( \Phi_1 - \Phi_2 - U_{\text{COR}}^0 \right) \right) \tag{123}\]

where the transfer coefficient is around 0.67, and the exchange-current density is on the order of \(5 \times 10^{-18} \text{A/cm}^2\) but with an activation energy of 134 kJ/mol. Thus, at high potentials and high temperatures the kinetics will become appreciable enough to drive the reaction. It is also known that the reaction rate is proportional to water partial pressure and logarithmic in time.

From the kinetic expression and associated rate constants, one expects that at higher potentials, > 1.2 V, carbon corrosion can proceed rapidly. Such high potentials have been shown to occur during start-up and shut-down or by local obstruction or starvation of hydrogen due to anode blockage or hydrogen depletion. This helps to explain why cells degrade more rapidly under cyclic conditions than they do when operated with fixed current densities and flows. Reiser et al. first explained the mechanism and presented a one-dimensional, steady-state model showing the development of large interfacial potentials in the hydrogen-starved regions. This reverse-current mechanism has been shown to exist, where potentials as high as 1.6 V have been measured when fuel and air coexist at the
fuel electrode during start-up/shut-down events.\textsuperscript{360} The mechanism involves a current that is produced in the normal part of the cell driving that in the starved region as shown in Figure 24. In the hydrogen-starved region, the cell potential remains constant (shown by the dotted lines) but the local anode and cathode potentials vary due to a drop in the electrolyte potential such that the negative side undergoes ORR and the positive side undergoes carbon corrosion and oxygen evolution. Due to the high level of carbon corrosion from the high potentials created during H\textsubscript{2}/Air fronts and localized anode fuel starvation, these high potentials have been employed as accelerated stress tests for the catalyst layer support.

![Figure 24. Potential distributions along anode flow path during reverse-current conditions (taken from reference 359).](image)

To model such effects,\textsuperscript{354,358,361} one only needs the general transport equations described in the background section along with the carbon-oxidation-reaction rate equation (123) and with a Butler-Volmer equation (57) that includes oxygen evolution,

\[ i_{\text{orr}} = i_{\text{orr}}^0 \left[ \exp \left( \frac{\alpha_{\text{orr}} F}{RT} \left( \Phi_1 - \Phi_2 - U_{\text{orr}} \right) \right) - \left( \frac{a_{\text{orr}}^\text{ref}}{a_{\text{orr}}^\text{ref}} \right)^4 \left( \frac{P_{\text{O}_2}}{P_{\text{O}_2}^\text{ref}} \right) \exp \left( -\frac{\alpha_{\text{orr}}^\text{ref} F}{RT} \left( \Phi_1 - \Phi_2 - U_{\text{orr}} \right) \right) \right] \quad (124) \]
which occurs simultaneously with carbon corrosion. To reduce the impact of the reverse-current mechanism, one can limit the gas transport through the membrane or short the stack, although in practice this is quite complicated because the hydrogen front does not reach each cell at the same time. Transient models that simulate the distribution of fuel from cell to cell during the introduction process are needed to optimize carbon-corrosion mitigation strategies.

Experimentally, carbon corrosion has been shown to have a number of effects on PEFC performance. These include changes to the electrocatalyst activity and kinetics, changing electrode-layer structure and porosity, and changing chemical surfaces which change the electrode-layer wetting properties (i.e., hydrophilicity); these combined effects are difficult to separate unambiguously and is an opportunity for modeling, especially these latter ones that directly impact the transport phenomena describe in this review. The associated effects of carbon corrosion on performance appear to depend upon many factors including the level and degree of carbon corrosion. As shown above, the carbon-corrosion process can be divided into two main reaction steps: (1) oxidation of the carbon surface to carbon-oxygen groups, and (2) further corrosion of the oxidized surface to carbon dioxide/monoxide. Carbon-surface oxidation increases the double-layer capacitance due to increased specific capacitance for carbon surfaces with carbon-oxygen groups. It may as well impact the wettability of the surface, making the carbon more hydrophilic and thus prone to flooding, which counteracts possible gains in porosity due to corrosion. Most studies, however, have demonstrated a loss in the cathode catalyst layer porosity, as shown in Figure 25, where approximately 50% of the porosity was lost within 10 hours of the accelerated stress test. Based on the electron microscopy observations of the centered-hollowed carbon particles and interconnected voids, it is concluded that carbon corrosion took place in an inside-out mode, with the carbon aggregate, rather than the carbon primary particle, as a basic corrosion unit. The size of carbon agglomerates decreased since carbon corrosion caused carbon
particles to decrease in size. As carbon corrosion proceeds, the ionomer to carbon ratio of agglomerates increase, thus exacerbating possible mass-transport losses associated with ionomer films; to date still little is published on the overall interaction between the carbon surface and the ionomer in the catalyst layer under degradation. Overall, carbon corrosion results in a collapse of the electrode’s porous structure and surface area, with loss of mass transport in the electrode, and subsequent degradation of fuel cell performance. This effect is more severe underneath the flowfield lands, and especially on the anode land area where local fuel starvation may occur. A complete understanding of the cathode catalyst-layer water saturation, how that changes with durability and operating conditions and how catalyst-layer degradation changes is not yet adequate for a full development of being able to predict mass transport in these layers.

Figure 25 TEM micrographs of a cathode catalyst layer (a) fresh, and after a carbon corrosion accelerated-stress-test hold at 1.2 V (b) after 10 hr and (c) 40 hr.

Finally, one should also mention the role of GDLs and MPLs on carbon corrosion, as they are integral for water management as discussed above. However, the exact effect that GDL/MPLs have on catalyst-
layer carbon corrosion has been difficult to ascertain. Potential-hold testing for acetylene-black fabricated MPLs resulted in dramatic loss of cell performance, and the introduction of a graphitized carbon in the MPL reduced the current density loss by > 50%.\textsuperscript{373} Omitting the cathode MPL increased the rate of kinetic loss but slowed the mass-transport losses, also suggesting that the MPL itself was degrading during the accelerated stress test.\textsuperscript{374} In general, cells with MPLs appear to be more resistant to prolonged carbon corrosion.\textsuperscript{375} As discussed above, the interface between a Pt/C cathode and MPL have been shown to be distinct, where the MPL carbon retains its meso-graphitic structure and porous network (even adjacent to the cathode surface) whereas the carbon support used for Pt is fully oxidized.\textsuperscript{376}

\textit{Platinum dissolution, migration, and redistribution}

Studies show that Pt can dissolve in acidic media at pH values representative of the electrolytes in PEFCs,\textsuperscript{377-379} especially at higher potentials.\textsuperscript{380} As the potential is cycled from high to low, equilibrium moves Pt into solution, and back out of solution causing Pt redistribution.\textsuperscript{381} This is depicted in Figure 26(a).\textsuperscript{382} Other mechanisms (depicted in the figure) which cause electrochemical active surface area loss include Pt particle migration (b), Pt particle detachment (c), and dissolution with precipitation in the ionomer phase (d). The effect of platinum redistribution leads to a loss in catalytic activity, reducing PEFC performance. The amount of Pt redistribution depends upon operating conditions including temperature, potential, RH, and presence of contaminants.\textsuperscript{383,384} Much of the loss of surface area is attributed to growth in platinum particle size in the catalyst layer. Coarsened platinum particles have been classified into two groups: spherical particles still in contact with the carbon support and nonspherical particles removed from the carbon support.\textsuperscript{385} The former results from electrochemical ripening, and the latter results from deposition in the ionomer by dissolved hydrogen. Both processes require preceding dissolution of the platinum. Pt dissolution/agglomeration can be diagnosed by a small
decrease in limiting capacitance (the extent of which will depend upon wt-% Pt) and small change in electrode resistance.$^{386}$

Figure 26. Schematic representation of platinum dissolution and redeposition (reproduced from reference $^{383}$).

The mechanism of Pt transport has been primarily shown to be in platinum cations. Both Pt(II) and Pt(IV) species have been detected in a sulfuric solution after potential cycling, and confirmed that the charge difference between anodic and cathodic sweep corresponds with the amount of dissolved species if the upper limit of the potential cycling is chosen to avoid oxygen evolution.$^{387}$ Pt is transported from the cathode catalyst, and where reprecipitation occurs seems dependent upon operating and
transport conditions. Much of the Pt reprecipitates in the cathode catalyst layer creating larger platinum particles as noted in Figure 26, and seen experimentally.\textsuperscript{388,389} Some of Pt also reprecipitates in the Nafion membrane, primarily where hydrogen and oxygen diffusion fronts meet and hydrogen reduces the platinum cations creating a ‘band’ of platinum.\textsuperscript{390-394} Platinum has also been shown to plate out on the anode catalyst layer/membrane interface.\textsuperscript{306,320}

To describe accurately platinum oxidation and dissolution requires inclusion of anodic and cathodic terms describing both the oxidation and the reduction of platinum oxide to platinum. For example, one can include equilibrium Pt-oxide coverage term in the kinetics (see equations 59 and 60).\textsuperscript{111,395,396} This coverage is potential dependent and ranges from zero to full coverage, where a full coverage protects the underlying Pt from dissolving even at higher potentials. Thus, one can see the reason why cycling is so detrimental is that one is transitioning to where Pt wants to dissolve thermodynamically and is not yet protected from doing so by the Pt-oxide layer. A more complicated expression can be used for Pt oxide formation, assuming that Pt forms a non-ideal solid solution with the Pt oxides that appear at potentials higher than 0.85 V.\textsuperscript{377} Thus, the activity of Pt in Pt-PtO\textsubscript{x} solid solution was derived from experimental dissolution data as

$$E = E^0(\text{Pt}) - \frac{RT}{nF} \ln(a_{\text{Pt}}) + \frac{RT}{nF} \ln(c_{\text{Pt}^{2+}})$$

(125)

where $c_{\text{Pt}^{2+}}$ is the concentration of Pt$^{2+}$ in the perchloric-acid solution. This model was expanded for use on catalyst-coated membranes and related to accelerated stress tests.\textsuperscript{397} The rate at which Pt dissolves by the reaction $\text{Pt} = \text{Pt}^{2+} + 2e^-$ is given by

$$i_{\text{Pt}^{2+}} = k \exp \left( \frac{\alpha \Theta_{\text{Pt}^{2+}}}{RT} \right) f(\phi) \left[ a_{\text{Pt}}(d_p, \Theta_{\text{Pt}O}, T) \exp \left( \frac{\alpha nF}{RT} (E - E^0(\text{Pt})) \right) - c_{\text{Pt}^{2+}} \exp \left( - \frac{\alpha nF}{RT} (E - E^0(\text{Pt})) \right) \right]$$

(126)
This model expansion includes the dynamics of particle-size evolution ($d_p$) considering particle growth by Ostwald ripening and coalescence/sintering. Results from this model indicate that the observed growth in particle size and loss in surface area are primarily due to coalescence/sintering and are sensitive to the potential limits and the initial particle-size distribution. An enthalpy of dissolution of 49.3 kJ.mol$^{-1}$ and an effective heat of fusion of 28.2 kJ.mol$^{-1}$ for particle coalescence/sintering have been empirically determined from the measured temperature dependence of particle growth. A systematic study is still needed to investigate the effect of O$_2$ on the thermodynamics and kinetics of Pt dissolution in aqueous media. Additionally, the model suggests that a reduction in ionomer hydration limits the diffusion of Pt ions in the ionomer; data is needed to estimate the platinum dissolution rate dependence on ionomer hydration.

As mentioned, Pt ions can move into and across the ionomer, which is a solid acid. This results in deposited particles in the membrane as well as the Pt band mentioned above. A substantial amount of Pt can be observed migrating into the membrane induced from potential cycling; measurements have shown up to ~ 13% to 20% of the cathode catalyst-layer Pt deposited into the Nafion membrane. A model based on dilute-solution theory (see equation 28) has been used to describe the movement of soluble platinum through the Nafion membrane.

There are few publications describing the platinum transport and platinum-dissolution effects on mass-transport limitations in PEFCs, although they seem to indicate larger mass-transfer overpotentials.

Other restructuring effects

Cathode catalyst-layer thinning is regularly observed and most reports indicate increases losses associated with mass transport. This catalyst-layer thinning is coupled with loss of porosity in the
catalyst layer, and in some cases correlate with carbon corrosion. It is unclear as to the causes of this catalyst-layer thinning, although restructuring and compaction with an associated loss of porosity of the catalyst layer is suspected. It is also unclear if this effect is widely observed for different MEA manufacturing techniques.

Results have indicated that structural changes in the PEM and catalyst layers are the main reasons for the decline in performance during open-circuit operation as well. The results also show that degradation due to Pt oxidation or catalyst contamination can be partially recovered by a subsequent potential-cycling process, whereas the same cycling process cannot recover the membrane degradation. Ionomer degradation in the catalyst layer and membrane have been assumed to be the cause of performance decline during open-circuit-potential holds.

Degradation during freeze/thaw testing have also been attributed to catalyst-layer structural changes and associated with the increased charge transfer and mass-transfer resistances, especially the internal diffusion resistance. Related are issues of catalyst-layer delamination, which will impact PEFC performance and transport modeling. To understand these effects, nascent efforts have focused on plasticity and mechanics models of the catalyst layer and its components. The cohesive zone model and the frictional contact model are used to describe the evolution of interfaces between the protonic and the electronic conducting phases.

Other catalyst-layer effects include dissolution of alloying materials to make platinum alloy. These alloying materials include other noble metals (e.g. Pd, Au, Ir), and metals including Ni, Co, Cr, Fe etc. The leaching effect of these alloying metals on catalyst-layer structure and transport remains widely unknown to date; most studies have concentrated on the electrocatalyst kinetics.
Future Perspectives

The above sections outline the critical areas of research and the current progress for continuum-scale modeling of transport in PEFCs. Throughout, not only background and current state of the art is discussed, but critical needs to move the field forward are mentioned. In this section, several of those needs are highlighted and discussed further. In addition, discussion is made on some underlying critical issues for the modeling community including the intersection between the models and experiments, accounted for variations in properties and structures, and the use of open-source modeling.

Intersection with experiments

As noted throughout this review, the modeling efforts depend heavily on understanding and elucidating the correct physics driving the observed phenomena. In addition, models rely on input parameters of the correct properties, and should be validated by both local and global cell performance and distributions of variables. We believe that to advance the art and understanding of transport modeling requires synergistic efforts in determining effective properties and providing model validation through systematic and orthogonal data sets. In other words, a model should use independently validated submodels or ex- or in-situ measured properties as input parameters, and then validated in terms of multiple in-operando observations (e.g., water content, response to operation changes, impedance spectra, polarization behavior, etc.), such as that provided at pemfcddata.org. If any fitting parameters are utilized, these should be fit simultaneously to the set of relevant experimental data. In this section, some of the key advances occurring or needed to occur will be highlighted including measurement of effective properties, cell-level studies, and electrochemical impedance spectroscopy.

Effective properties
Recently, there has been a substantial effort to measure directly many of the key transport properties as a function of the various operating conditions such as temperature, pressure, compression, relative humidity, and liquid saturation. Typically, these are measured ex situ, and care must be undergone in translating results for complete layers into the discretized modeling domain as mentioned in the multiphase section above. In addition, more sophisticated techniques are being used to validate in situ measurements that were previously accomplished ex situ. For example, recent neutron imaging experiments have confirmed that the measured membrane water content in an operating PEFC is similar to the ex-situ measurements reported over two decades ago.\textsuperscript{33,60,65} Also, in terms of transport properties, methods have been developed to separate the GDL mass transport from the catalyst-layer transport as well as diffusion mechanisms by varying inlet gas compositions and temperature and total pressure.\textsuperscript{37,270,405-407}

Many of the recent studies have focused on multiphase flow and developing diagnostics to measure effective properties as a function of saturation. For example, one is now using capillary pressure – saturation curves that are measured to calibrate pore-network models that then can simulate different structures and be used for parametric studies, as discussed above. Similarly, there are studies on the perhaps more important variables of breakthrough pressure\textsuperscript{408-410} and droplet wettability and adhesion, both internally and externally.\textsuperscript{130,203,411-415} Also, researchers have begun to characterize how the resulting saturation impacts the thermal resistivity. Ex-situ investigations of thermal resistance as a function of saturation have shown that this relationship can be accurately approximated by an effective thermal conductivity.\textsuperscript{416,417} In terms of oxygen diffusivity, which is a key quantity impacting performance, in-situ characterization has been investigated with AC impedance spectroscopy,\textsuperscript{418} conductivity cells,\textsuperscript{40} mercury-intrusion porosimetry,\textsuperscript{129,419} x-ray computed tomography (XCT),\textsuperscript{41} Loschmidt diffusion cells,\textsuperscript{42} and electrochemical techniques.\textsuperscript{43} As discussed in the empirical-modeling section, Baker\textit{ et al.}\textsuperscript{37} and
Caulk et al.\textsuperscript{44} established a method and analysis using limiting-current measurements under dry and oversaturated conditions. These limiting current methods and the corresponding analysis have shown that the effective diffusion coefficient follows a characteristic behavior. With increasing current density, a dry then a wet plateau is observed for several types of carbon fiber GDLs.

As discussed in the catalyst-layer section, the catalyst layer is the least characterized of the PEFC components due to the difficulty in measuring its properties. Recent work has concentrated on measuring the properties of the catalyst layers on various polymer, metallic, and non-metallic substrates. The permeability and porosity of a catalyst layer supported on ethylene tetrafluoroethylene has been determined and the catalyst porosity measured using traditional mercury-intrusion porosimetry is lower that what would be expected from typical catalyst layers.\textsuperscript{420} The water uptake in catalyst layers supported on PTFE membranes has been reported to be lower than that expected from bulk ionomer measurements,\textsuperscript{131} which directly agree with studies of ionomer thin films as discussed above. The ionic conductivity of PEFC catalyst layers can be directly measured and modeled using AC impedance techniques\textsuperscript{278} and has been found to be \(\approx 100 \text{ m}\Omega \text{cm}^2\) in state of the art Pt/C electrodes,\textsuperscript{278} which is in conflict with some of the thin-film measurements;\textsuperscript{292,304,308,421} there is no current agreement. The ionic conductivity is a strong function of the Nafion\textsuperscript{*} content and needs to be optimized for optimal ionic conductivity and gas transport.\textsuperscript{422,423}

**Visualization and validation**

The modeling results have traditionally been validated using PEFC performance data, mainly in the form of polarization curves.\textsuperscript{22,23} However the insensitivity of PEFC polarization curves to varying component materials properties and operating conditions makes model validation a challenge.\textsuperscript{424} To this end, there is a need for validation through comparison of local variables.
Segmented-cell data has better fidelity than single polarization curves. Segmented cells have been utilized extensively as a diagnostic tool for PEFC as recently reviewed. The most widely used segmented-cell techniques (commercially available systems) involve segmenting the current collector and measuring the resultant current using printed circuit boards. There are also several groups that utilize segmented GDLs and even segmented catalyst layers with current being measured either by resistor networks or Hall-effect sensors. In addition to measuring current, some systems allow simultaneous measurement of cyclic voltammograms and EIS spectra in each of these segments.

Direct visualization of liquid water in PEFCs is an excellent tool for model validation and significant progress has been made in the past decade in improving visualization techniques. These techniques include direct optical imaging of specially designed transparent PEFCs, x-ray radiography and tomography, neutron scattering, radiography, and tomography, and NMR imaging. For catalyst layers, while recent work has demonstrated that nano X-ray computed tomography can be utilized to measure their pore distribution, it is challenging to obtain the ionomer. Scanning transmission x–ray microscopy (STXM), scanning electron microscopy and other microscopy techniques are being developed to better visualize catalyst-layer structures. It should also be noted that these techniques normally involve image-based reconstructions of small areas. Thus, there is a need to address the lack of statistic-based variability and the cost/time expense of imaging a sufficient number of samples to attain reproducible results, especially for simulations or validation that depend on them. With the current analysis techniques, the simulation cost from a time perspective is still inherently prohibitive if sample domains approach the scales seen in real PEFC systems. From a validation standpoint, phase resolution in the diagnostic imaging and the use of phase-differentiated distribution data would yield a boon in increasing the confidence in the design capability of these methods.
Amongst the various visualization techniques, the only one capable of imaging water \textit{in operando} in large-area PEFCs without the need to change MEA, bipolar-plate, and end-plate materials is neutron imaging.\textsuperscript{429} Although in-plane neutron imaging has been extensively utilized to image water in the PEFC flowfields,\textsuperscript{439-441} there is not a clear correlation between flowfield water and PEFC performance. The flowfield water observed by neutron imaging has only been successfully correlated to the pressure drop observed and is a useful tool for flowfield and manifold design.\textsuperscript{441}

The resolution of neutron-imaging techniques have improved dramatically over the past decade and high-resolution neutron imaging has been utilized to quantify the through-plane water distribution in PEFCs.\textsuperscript{65} The advantage of high-resolution through-plane imaging is that it has the potential to provide unambiguously the liquid-water content within the MEA components of an operating PEFCs,\textsuperscript{134,442-444} and can be utilized to validate PEFC models.\textsuperscript{65,134} In terms of the last point, Weber and Hickner\textsuperscript{134} directly compared simulation results to measured water contents, and highlighted PCI flow in the GDLs and discrepancies between the two data sets that still need to be reconciled. Through-plane imaging has also been utilized to demonstrate the water profiles inside the MEAs with different GDL compositions, illustrating the effect of the MPL on cathode-catalyst-layer water content.\textsuperscript{445} Hussey \textit{et al.}\textsuperscript{65} addressed various systematic errors in high-resolution neutron imaging to determine accurately the water content in thick Nafion\textsuperscript{®} membranes (up to 1000 \(\mu\)m thick). This work clearly demonstrated that high-resolution neutron imaging can accurately quantify \textit{in-situ} water content in thicker PEFC components like GDLs and thick membranes. However, the state-of-the-art in imaging still requires improvement in order to quantify catalyst-layer and membrane water content in commercial MEAs.

\textbf{Electrochemical impedance spectroscopy}
Electrochemical impedance spectroscopy (EIS) is a powerful technique used to understand complex phenomena occurring in a PEFC cell. This technique is closely associated with catalyst-layer phenomena since it allows exploration of dominant resistances that occur typically in the catalyst layer and requires electrodes to get a response. The application of EIS to PEFCs has been illustrated in detail in books, book chapters, and review articles. While other techniques are readily modeled with the various modeling equations or derivatives thereof, EIS falls into a special category due to its pervasiveness, ease, and mystery in understanding and interpreting its resulting data, which are compounded by the complexity of the overlapping phenomena.

Impedance spectroscopy is a perturbation technique. The idea is that by applying only a small perturbation during operation, the system response can be studied noninvasively in operando. A sinusoidal perturbation is applied to a system operating under steady state and its response is monitored. Depending on the rate of the processes involved in the system, the perturbed variable is transmitted across the system. So, for a voltage oscillation of

\[ E_i = E_0 \sin(\omega t) \]  

(127)

the current response is phase shifted by \( \varphi \) to be

\[ I_i = I_0 \sin(\omega t + \varphi) \]  

(128)

and the frequency-dependent EIS is given in terms of real and imaginary parts

\[ Z(\omega) = \frac{E}{I} = Z_0 (\cos \varphi + j \sin \varphi) \]  

(129)

The highly non-linear nature of PEFC polarization requires that EIS be performed at various potentials along the polarization curve and then fitted with either an equivalent circuit model or more complicated physics-based model. However, in PEFCs, there are very few publications that
use these latter realistic physical models to describe experimental EIS data, and this is a glaring need for
the community since EIS is a ubiquitous technique that has the potential to provide a wealth of property
and validation data.

To model the impedance, one can follow the methodology of Newman and coworkers. From a
physics-based model, each of the dependent variable can be written as the sum of its steady-state
component (−) and time-varying component (~) that is frequency dependent, for example current is
written as,

\[ I = \bar{I} + \tilde{I}_i \]  \hspace{1cm} (130)

The time dependent component for a sinusoidal perturbation with fixed amplitude is

\[ \tilde{I}_i = I_0 \sin(\omega t) \]  \hspace{1cm} (131)

Hence the net current is,

\[ I = \bar{I} + \text{Re}\{\tilde{I}_i e^{j\omega t}\} \]  \hspace{1cm} (132)

Similarly, for any variable in the governing equations described in the previous sections, we write

\[ x_i = \bar{x}_i + \text{Re}\{\tilde{x}_i e^{j\omega t}\} \]  \hspace{1cm} (133)

To determine the frequency dependent part of each of the variable, each of the variables is expanded in
a Taylor series. Assuming the perturbation is small enough that the system responds linearly, the higher-
order terms are neglected,

\[ f(x) = f(\bar{x} + \text{Re}[\tilde{x} e^{j\omega t}]) = f(\bar{x}) + \left. \frac{df}{dx} \right|_{\bar{x}} \text{Re}[\tilde{x} e^{j\omega t}] \]  \hspace{1cm} (134)

Rewriting the above equations in matrix form to evaluate the unknown frequency dependent terms,
\[
\begin{bmatrix}
J_{\text{real}} & -J_{\text{im}} \\
J_{\text{im}} & J_{\text{real}}
\end{bmatrix}
\begin{bmatrix}
\tilde{\nu}_{\text{real}} \\
\tilde{\nu}_{\text{im}}
\end{bmatrix}
= \begin{bmatrix}
G_{\text{real}} \\
G_{\text{im}}
\end{bmatrix}
\]  \hspace{1cm} (135)

Thus, there is only the need to evaluate the time derivative and one can make use of the existing Jacobians to speed up the solution process. From the resulting values, the frequency-dependent EIS is written as the ratio of the frequency-dependent potential and current,

\[Z = \frac{\tilde{\nu}}{i}\]  \hspace{1cm} (136)

By using the governing equations and expressions, one can understand how each variable or property affects the EIS. In terms of computation, the transformations can be done numerically or analytically if possible, and essentially the number of unknowns doubles since each variable now has both a real and an imaginary component.

Overall, EIS is a very powerful experimental tool, especially for characterization and trends, but its results are only as meaningful as the model used for its analysis. Some EIS data is relatively simple to understand such as the high-frequency resistance, which is the sum of the membrane resistance, contact resistances, and electronic resistance in the system, and can also be utilized to calculate the membrane-electrode interfacial resistance. More complicated analysis arises as one uses EIS to explore the ionic conductivity within the catalyst layer, where the data are often fit using a modified transmission-line model originally proposed by Lefebvre. The high-frequency arc in the EIS spectrum consists of a 45° angle line in addition to a capacitive loop and represents the coupled ORR kinetics and catalyst-layer ionic resistance. This high-frequency loop can be modeled to determine the ORR kinetic parameters including exchange current density, Tafel slope and reaction order. Two Tafel slopes have been observed from the EIS spectra, one at low current densities corresponding to a PtO surface and another at higher current densities corresponding to a reduced Pt surface. The EIS
results are consistent with the two Tafel-slope model proposed in the literature\textsuperscript{464} and do not support the single Tafel-slope model.\textsuperscript{97,103} These EIS measurements assume that the anode reactions are significantly faster than the cathode reactions and the H\textsubscript{2} electrode serves as a pseudo reference electrode.

Finally, interpreting the low-frequency part of the EIS response is the most difficult and least understood as it pertains to mass transport and many overlapping phenomena both within the cell sandwich and along the channel.\textsuperscript{418,452,465} For example, to date, the mass-transfer resistance measured with EIS has not been directly correlated to the saturation measured with visualization techniques. Also, the low-frequency (DC) EIS resistance values are significantly lower than that observed from the slope of the polarization curve at that current density, indicating that a low-frequency inductive loop should be taken into account in any PEFC model that attempts to simulate EIS data.\textsuperscript{466} Such low-frequency inductive loops have been observed in EIS spectra.\textsuperscript{278,466} This inductive behavior has been attributed to adsorbed intermediate oxygen species related to the ORR\textsuperscript{461} or to effects of fluctuating water concentration along the channel.\textsuperscript{455,467,468} Schneider et al. performed EIS on a segmented cell combined with in-situ neutron imaging and clearly demonstrated the absence of this arc in those sections of the cell that were exposed to liquid water.\textsuperscript{468} A comprehensive model that takes into account liquid water and is directly able to fit a wide variety of impedance spectra obtained from a cell at different operating conditions is still lacking. However, simpler equivalent-circuit models have been effective in qualitatively correlating mass-transport limitations and providing directions to improve gas transport and PEFC performance.

\textit{Modeling stochastic and statistical performance}
As discussed in early sections in terms of this review, there is a variability to experiments that should be represented in modeling. For example, freeze kinetics is based on statistical experimental data, as are virtual reconstructions of various cell layers and statistical-based models like pore-network modeling. In addition, experiments also show natural variability due not only to the component variability but also to fluctuations in operating conditions, etc. Models should account for these variations in terms of detailed sensitivity studies, and, for the case of stochastic simulations, an adequate number of virtual experiments or structures.

Given that most of the materials within the MEA present themselves as heterogeneous porous media and that the few that do not still demonstrate variation geometrically from their respective manufacturing processes, it becomes of interest to study the numerical prediction of the cell from a similar statistical basis. This process is relatively straightforward as it is the numerical inputs that are determined prior to a simulation via a statistical selection process. By approaching the inputs in this way, the subsequent simulation represents a “given” configuration out of a batch of \( N \) configurations, with \( N \) being selected to achieve a desired confidence interval in the performance data. As an example of the type of statistical variation that can be used, Figure 27a shows a typical variation of thickness (geometrical) variation of a GDL as determined by statistical parameters determined from experimental measurement of a batch of GDLs that were to be used in model validation. The figure shows the use of the statistical parameters in creating a “run-to-run” variation of GDL thickness. The use of statistical analysis like this dramatically alters the manner by which a model is said to be validated, as in physical reality one would not achieve a perfect repeat of experimental measured performance curves even if re-testing the exact same PEFC with even greater variation being seen if a similarly configured, but new, PEFC was used.
Figure 27b shows the results of a validation of a performance model from a statistical basis; from this figure it is interesting to note several aspects. One of the first things is that the statistical variation of the experiment widens as the performance increases, this is of interest as it implies that the cell is becoming increasingly sensitive to various aspects of the material properties or manufacturing variations and that stricter tolerances or controls on the porous-media properties may be required depending on the desired cell operating point. Furthermore, one should also notice that there are combinations of the material characteristics that result in the formation of the characteristic “performance knee” in the 1.2 to 1.4 A/cm² region where other combinations do not show this and maintain a nearly linear behavior. From the inputs, this aspect seems to be at least in part derived from multiphase flow phenomena. In summary, given the statistical nature of the materials used and the statistical nature of the performance data itself, it becomes apparent that validation of and any derived from performance or durability simulations should be done using some type of statistical basis.
Open-source models

With the advent of open-source publishing and looking towards the future, this section describes the recent efforts for open-source fuel-cell models and also a future outlook of its possibilities. The operation of PEFCs involves many coupled physical phenomena as discussed above, and therefore the development of mathematical physical models is a very challenging endeavor. It is for this reason that over the past two decades there has been tremendous growth on the number and complexity of PEFC models. The majority of PEFC models to date have either been implemented in commercial software
such as FLUENT, COMSOL, STAR CD, or implemented in-house. In either case, the source code has not been made available to the public. This has several major drawbacks including:

- **Lack of validation and comparison between models.** To validate previous models, an independent research group would have to spend a considerable amount of time to redevelop the model. The result is that many of the mathematical models currently in the literature have not been thoroughly validated. Furthermore, most models are not compared to one another.

- **Lack of extension capabilities.** PEFC mathematical models are constantly evolving. Therefore, previous mathematical models need to be extended to include the most recent understanding of physical processes, such as those discussed above. Only if the source code is available can previous models be extended by other scientists in an efficient and ready manner.

- **Implementation limitations.** When using commercial software, even if the source code is available, it is not always possible to extend the physics, especially if the partial differential equation does not meet some standard form.

Due to the drawbacks mentioned above, each research group usually re-implements an improved version of the previous mathematical model proposed in the literature, thereby reinventing the wheel and wasting precious time that could be used implementing new features. Furthermore, since each research group includes a modification to the previous model in the literature, and the previous model is not available to the new researchers, mathematical models are not usually compared, thereby making it impossible to assess the impact of the proposed modifications. For example, different versions of the catalyst-layer agglomerate model have been implemented at least a dozen times in the past.
however, a detailed comparison between the different models, i.e. using the same geometry and parameters for the different implementations, has yet to be performed.

To enhance collaboration between researchers, several research groups have recently started to share their research source codes, which is believed to be a key future issue/opportunity. To date, there are three known open-source packages in the literature, i.e. the code developed by Novaresio et al.\textsuperscript{472}, openFuelCell,\textsuperscript{473} and openFCST.\textsuperscript{474} The first two codes focus on solid-oxide fuel cells (SOFCs). The latter focuses on PEFCs. Furthermore, the former two codes are based on the openFOAM finite volume libraries\textsuperscript{280} while the latter is based on the deal.II finite element libraries.\textsuperscript{475}

The main advantages of developing, sharing and using open-source codes can be summarized as:

- **Transparency.** Researchers can provide the source code and data files with their articles so that other scientists can verify their results

- **Ease to develop new routines** by using already existing classes/functions

- **Ease to share new developments** with the fuel-cell community

- **Ease of integration.** It is easy to integrate new mathematical models with already existing models in order to develop complex, multi-dimensional fuel cell simulations within a short timeframe

- **Easy to validate and compare to previous models**
  
  - New models can easily be compared to previously implemented models under the same architecture.
If a large user base is established, there will be a large number of testers. This would make bugs in the code easy to find.

It is easy to track and fix bugs. For example, both openFCST and openFuelCell have a ticketing system so that users can submit bugs that were found in the code.

- **Ease of integration with other programs.** For example, all the open source codes discussed above are integrated with other open-source packages and algorithms such as openFOAM, Trilinos, deal.II, and Dakota.\(^{283}\)

The three open-source packages mentioned above were released in 2012 and 2013; therefore, at this point, none of the packages have a large user base. Part of the reason for the slow uptake of open-source codes is that they require a steeper learning curve than commercial software. In the case of in-house codes, the learning curve is worse than open-source packages since most open-source packages provide extensive documentation, e.g., openFCST contains a user guide, a developer guide and a class list reference guide in HTML.\(^{474}\) The key disadvantages of most open-source codes are no graphical user interface and a necessary knowledge of Linux OS and a programming language (preferably an object-oriented language, e.g. C++ or python). To reduce the steep learning curve, most open-source codes have extensive documentation, a mailing list that is used to help new users get started, and many examples.

Due to the advantages of sharing and working together on codes, it is believed that the fuel-cell scientific community could benefit from an open-source framework. The new routines developed by users can be integrated with the current open-source packages thereby leading to the required level of sophistication needed in order to increase our understanding of the physical processes taking place in PEFCs. To overcome the key disadvantages of most open-source packages is to hold trainings and get a
buy-in from the community in order to generate a large enough user base. One possible mechanism might be the formation of a clearinghouse where codes can be developed, validated, and enhanced. In this fashion, the main contributing institutions could host a welcome page, the main repository, testing site, and mailing list that all developers submit questions and bugs to in order to ensure a large user base and subsequent quick response times and discussions. In addition, comparison of the simulation results to standard data sets such as the provided at pemfcdato.org using consistent and standard input parameters can provide insights and useful measures of operating ranges and need for better models in terms of properties and phenomena.

As discussed throughout this review, although there are some contentious issues, most of the PEFC physics is becoming standardized. In addition, most PEFC codes are implemented with either a finite-volume or a finite-element analysis library already as a backbone, and thus transitioning to open source may not be that problematic. While full open source is perhaps a good end goal, an intermediate goal of use of a common commercial software package and sharing of the model files could also be a means to bring together the community to build around a single model set. This would not be as easy to modify as open-source code, but it would allow for greater accessible of the simulations to the larger fuel-cell community, which include the casual modeler. This also alleviates the steep learning curve for open-source codes, especially if the community also provides input into the methods to make the commercial package converge and solve.

Summary and outlook

Modeling of polymer-electrolyte fuel cells (PEFCs) has advanced significantly over the last couple of decades, where the simultaneous advent of increased computational power and diagnostic techniques have allowed much greater predictability, complexity, and usefulness of the models. Today, design and
optimization strategies can be computationally explored with high fidelity and confidence. This critical review has focused on examining these advancements as they relate to modeling of transport phenomena in PEFCs. Throughout, we have discussed the governing phenomena and equations while highlighting the needs and gaps. Of particular focus are issues surrounding multiphase flow and catalyst layers. For the former, there are still unknowns in terms of the correct treatment of interfaces, and, in particular, that with the GDL/gas channel. These interfaces are seen as significantly influencing the overall transport and cell behavior. The land-to-GDL interface is characterized by relatively high ohmic and thermal resistance in conventional channel|land designs, and a location for accumulation of condensing vapor drawn by PCI flow. The channel wall-to-diffusion medium interface has been demonstrated to be a key factor in the retention of water in the DM and cell. The shape and surface condition is important to draw water out of the DM and prevent flooding in the DM. The GDL/MPL interface is a region that has so far been studied the least, with properties that are influenced by the manufacturing method and degree of co-mingling of the layer material. Particularly at the micrometer-scale, more work needs to be done to improve our overall understanding of the interfaces, so that all relevant effects can be included in state-of-the-art multiphase models.

For catalyst layers, there is a need to correlate better the phenomena at the local scale where ionomer thin-films or lack thereof impact performance with the phenomena at the macroscale and thus be predictive. For example, due to the heterogeneous porous structure of the catalyst layer, it is important to establish accurate correlations between the catalyst-layer ionomer and thin-films of ionomer on substrates. It is particularly critical to understand how catalyst-layer properties including, but not limited to, ionomer content, I/C loading, Pt loading, porosity could be studied in thin-film model systems using parameters such as film thickness, substrate/film interactions, film casting and processing.
Similarly, one needs to understand how processing conditions, such as hot pressing, can impact overall transport properties and behavior of the ionomer and catalyst layer.

As discussed throughout, there is still a need to link the macroscopic observables and equations with those at smaller length scales in order to provide a truly representative simulation. While such linkages are typically accomplished by transfer of properties, adaptive mechanisms and bidirectional coupling and multiscaling provide still new opportunities and challenges for modeling transport in PEFCs. Virtual or image reconstructions may allow for such a route, but there is still a need for better resolution and more knowledge of the interrelated conditions and formation rules, especially for catalyst layers.

Other transport modeling challenges remain. For example, one needs to understand the sensitivity of the various input parameters (e.g., transport properties) whether they come from experiment or lower-scale models. By knowing the sensitivity, we can see how that matches with experimental results and also provide direction to future activities and research areas. Similarly, this can help to link the modeling results with real-world results in which the systems are much more dynamic and variables with nonuniform properties and stressors. Similarly, one needs to analyze and understand the impact of defects and significant spatial variation in properties that can impact overall performance and durability. This is something that has only minimally been studied in the modeling literature.476,477

In the same fashion, in order to accelerate the components development it is necessary to correlate the measurable properties to the performance effect on a PEFC over its lifetime. This is a place where modeling has a possible significant role to play, since experiments require substantial times to enact. Also, the use of accelerated stress tests and their applicability to real-world and in-operando conditions can be correlated through validated cell models. The issue of lifetime predictions dovetails into that of durability modeling, where there is substantial room for improvement and understanding. While there
are some degradation specific models for phenomena as discussed such as catalyst dissolution, a majority of models just alter their transport or related (e.g., water-uptake isotherm or capillary pressure – saturation relationship) properties as a function of time. This method is similar to the use of a fitting parameter with time, and more in-depth modeling and knowledge of degradation phenomena are required to increase the usefulness of macroscopic modeling for durability as it is already for performance.

Overall, significant progress has been made and consensus is being reached for the general treatments and governing equations for modeling transport phenomena in PEFCs. However, as highlighted in this critical review, there are still some significant issues that require more research and understanding. As modeling becomes more common place, there is a need to ensure that the proper physics is elucidated and considered, and new modeling methodologies, treatments, and emphasis guided to the key issues.

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Notation

**English**

- $a_i^\alpha$ - activity of species $i$ in phase $\alpha$
- $a_{k,p}$ - interfacial surface area between phases $k$ and $p$ per unit volume, 1/cm
- $a^\circ_{1,2}$ - interfacial area between the electronically conducting and membrane phases with no flooding, 1/cm
- $a_{\text{film}}$ - specific surface-area of the film, 1/cm
- $A_{\text{agg}}$ - specific external surface area of the agglomerate, 1/cm
- $A_{\text{pore}}$ - cross-sectional area of pore, cm$^2$
- $A_{Pt}$ - reactive surface area of platinum, cm$^2$/g
- $b$ - Tafel slope, V
- $c_{i,k}$ - interstitial concentration of species $i$ in phase $k$, mol/cm$^3$
- $c_{Pt^{2+}}$ - the concentration of Pt$^{2+}$ in perchloric-acid solution, mol/cm$^3$
- $c_T$ - total solution concentration or molar density, mol/cm$^3$
- $C_p$ - heat capacity of phase $k$, J/g-K
- $C_d$ - differential double-layer capacitance, and diffuse layer capacitance, F/cm$^2$
- $C_H$ - Stern layer capacity, F/cm$^2$
- $D$ - the droplet diameter at its maximum, cm
- $d_i$ - driving force per unit volume acting on species $i$ in phase $k$, J/cm$^4$
- $d_p$ - particle-size, cm
- $d_w$ - wetted diameter, cm
- $D_i$ - Fickian diffusion coefficient of species $i$ in a mixture, cm$^2$/s
- $D_S$ - capillary diffusivity, cm$^2$/s
- $D_{i,j}$ - diffusion coefficient of $i$ in $j$, cm$^2$/s
- $D_K$ - Knudsen diffusion coefficient of species $i$, cm$^2$/s
- $E$ - effectiveness factor
- $E_t$ - voltage signal in EIS, V
- $f_{r,k}$ - the fraction of total distribution made up of distribution $k$
- $F$ - Faraday's constant, 96487 C/equiv
\(F_a\) - adhesion or surface-tension force, N
\(F_p\) - pressure force, N
\(F_s\) - shear force acting on the froplet, N
\(F_g\) - gravitational force, N
\(g\) - acceleration due to gravity, cm/s\(^2\)
\(\Delta G\) - Gibbs free energy of reaction \(l\), J/mol
\(h\) - droplet height, cm
\(h_{k,p}\) - heat-transfer coefficient between phases \(k\) and \(p\), J/cm\(^2\)s-K
\(H\) - channel height, cm, and Henry’s constant
\(H^f_i\) - heat of formation of species \(i\), J/mol
\(\overline{H}_{i,k}\) - partial molar enthalpy of species \(i\) in phase \(k\), J/mol
\(H_{ij}\) - Henry’s constant for species \(i\) in component \(j\), mol/cm\(^3\)kPa
\(\Delta H_l\) - heat or enthalpy of reaction \(l\), J/mol
\(\Delta H_f\) - heat of fusion of ice, J/mol
\(i\) - current density, A/cm\(^2\)
\(i_k\) - current density in phase \(k\), A/cm\(^2\)
\(i_{0h}\) - exchange current density for reaction \(h\), A/cm\(^2\)
\(i_{h,k,p}\) - transfer current density of reaction \(h\) per interfacial area between phases \(k\) and \(p\), A/cm\(^2\)
\(i_{lim}\) - limiting current density, A/cm\(^2\)
\(\overline{I}\) - steady-state current component in EIS, A
\(\overline{I}_t\) - time-varying current component in EIS, A
\(I_t\) - current signal in EIS, A
\(J_{i,k}\) - flux density of species \(i\) in phase \(k\) relative the mass-average velocity of phase \(k\), mol/cm\(^2\)s
\(J(s)\) - Leverett \(J\)-function
\(J(T)\) - pseudo-steady-state nucleation rate, 1/m\(^3\)s
\(k\) - effective hydraulic permeability, cm\(^2\)
\(k'\) - ORR rate constant, 1/s
$k^*$ - ORR rate constant, cm/s
$k_{evap}$ - evaporation rate constant, mol/cm$^2$s
$k_f$ - rate constant for ice formation, mol/cm$^2$s
$k_{O_2,film}$ - rate constant for oxygen dissolution, mol/cm$^2$s

$k_T$ - thermal conductivity of phase $k$, J/cm$^2$K
$k_r$ - relative hydraulic permeability
$k_{int}$ - mass-transfer coefficient, mol/cm$^2$s
$k_{sat}$ - saturated hydraulic permeability, cm$^2$
$k_{\Phi}$ - electrokinetic permeability, cm$^2$

$K_{i,j}$ - frictional interaction parameter between species $i$ and $j$, Ns/cm$^4$

$l$ - length of the pore between nodes $i$ and $j$, cm
$L$ - catalyst layer thickness, cm

$m$ - parameter in polarization equation
$m_{Pt}$ - loading of platinum, g/cm$^2$

$M_i$ - molecular weight of species $i$, g/mol

$M_i^{Z_i}$ - symbol for the chemical formula of species $i$ in phase $k$ having charge $Z_i$

$n$ - parameter in 0-D model equation for polarization curve

$n_h$ - number of electrons transferred in electrode reaction $h$

$n_{H^+}$ - the centerline number density of ions, 1/m$^3$

$N_{i,k}$ - superficial flux density of species $i$ in phase $k$, mol/cm$^2$s

$N_{N,H^+}$ - the density of charge droup within membrane, 1/m$^3$

$p_0^{vap}$ - saturation vapor pressure, kPa

$p_0^{vap,0}$ - uncorrected vapor pressure of water, which is function of temperature, kPa

$p_i$ - partial pressure of species $i$, kPa

$\Delta p_{ij}$ - pressure acting across the pore between nodes $i$ and $j$, kPa

$p_c$ - capillary pressure, kPa

$p_{Ci}$ - capillary pressure in the pore when multiple phases are present, kPa

$p_k$ - total pressure of phase $k$, kPa
\( P_{O_2,ig} \) - the equivalent pressure within the ionomer at the interface with gas, kPa

\( p_w^{\text{vap}} \) - vapor pressure of water, kPa

\( PZC \) - potential of zero charge, V

\( q_k \) - superficial heat flux through phase \( k \), J/cm\(^2\)s

\( q \) - charge in the double layer, C/cm\(^2\)

\( q_{w,pore} \) - volumetric flowrate of water cm\(^3\)/s

\( Q \) - total amount of heat generated, J/cm\(^2\)s

\( Q_{k-p} \) - heat flux transferred between phases \( k \) and \( p \), J/cm\(^3\)s

\( r \) - pore radius, cm

\( r_{agg} \) - radius of agglomerate, cm

\( r_c \) - critical pore radius, cm

\( r_{ij} \) - radius of cylindrical pore between nodes \( i \) and \( j \), cm

\( \bar{r}_i \) - average pore radius around the node \( i \), cm

\( r_{evap} \) - rate of evaporation, mol/cm\(^3\)s

\( r_{o,k} \) - characteristic pore size, cm

\( R_{l,k-p} \) - rate of reaction \( l \) per unit of interfacial area between phases \( k \) and \( p \), mol/cm\(^2\)s

\( R \) - ideal-gas constant, 8.3143 J/mol-K

\( R_{agg} \) - agglomerate radius, cm

\( R_{g,k} \) - rate of homogenous reaction \( g \) in phase \( k \), mol/cm\(^3\)s

\( R_{i,j} \) - resistance of resistor \( i,j \), \( \Omega \) cm\(^2\)

\( R_I \) - ice-crystallization rate, mol/cm\(^2\)s

\( R_i \) - rate of ice formation, mol/cm\(^2\)s

\( R' \) - total ohmic resistance, \( \Omega \) cm\(^2\)

\( R_{oxygen,film} \) - transport resistance of oxygen through ionomer film, s/cm

\( \text{Re} \{ \} \) - the real portion of time-dependent current in EIS

\( \mathcal{R}_{i,k} \) - total rate of reaction of species \( i \) in phase \( k \), mol/cm\(^3\)s

\( s_{i,k,l} \) - stoichiometric coefficient of species \( i \) in phase \( k \) participating in reaction \( l \)

\( s_k \) - spread of distribution \( k \)
$S$ - liquid saturation
$S^*$ - pore-size distribution
$S_e$ - effective saturation
$\bar{S}_{i,k}$ - molar entropy of species $i$ in phase $k$, J/mol-K
$S_L$ - equilibrium liquid saturation
$S_m$ - other sources of momentum in Navier-Stockes equations, N/cm$^3$
$\Delta S_h$ - entropy of reaction $h$, J/mol-K
$S_r$ - residual saturation
$t$ - time, s
$T$ - absolute temperature, K
$\bar{T}_c$ - number-average crystallization temperature, K
$T_{FPD}$ - the amount of freezing-point depression, K
$T_k$ - absolute temperature of phase $k$, K
$T_m$ - melting (freezing) temperature for bulk water, K
$u_i$ - mobility of species $i$, cm$^2$mol/J-s
$U_h$ - reversible cell potential of reaction $h$, V
$U'$ - potential intercept for a polarization equation, V
$U_h^0$ - standard potential of reaction $h$, for oxygen reduction, 1.229 V at 25°C
$U_{H_h}$ - enthalpy potential,
$v_k$ - superficial velocity of phase $k$, cm/s
$v_{w,pore}$ - water velocity in cylindrical pore, cm/s
$\langle v \rangle$ - average flow velocity in the channel, cm/s
$V$ - cell potential, V and droplet volume, cm$^3$
$V_i$ - (partial) molar volume of species $i$, cm$^3$/mol
$W_{O_2}^{\text{diff}}$ - molar flow rate of oxygen to the agglomerate, mol/cm$^3$s
$x$ - distance across the flow field, cm
$x_{i,k}$ - mole fraction of species $i$ in phase $k$
\( y \) - distance along the flow-field channel, cm
\( z \) - distance across the cell sandwich, cm
\( z_i \) - valence or charge number of species \( i \)
\( Z(\omega) \) - frequency-dependent, resistance in EIS, \( \Omega \)

**Greek**

\( \alpha_a \) - anodic transfer coefficient
\( \alpha_c \) - cathodic transfer coefficient
\( \alpha_L \) - liquid thermal diffusivity, m\(^2\)/s
\( \alpha_w \) - water transport coefficient, mol\(^2\)/J-cm-s
\( \beta \) - net water flux per proton flux through the membrane, and is the heating rate, K/min
\( \gamma \) - surface tension, N/cm
\( \delta_{film} \) - ionomer film thickness, cm
\( \delta_n \) - diffusion length or thickness of region \( n \), cm
\( \zeta \) - characteristic length, cm
\( \theta_{o,n} \) - characteristic contact angle of distribution \( n \)
\( \Theta_{PtO} \) - coverage of Pt oxide
\( \theta_s \) - the sliding angle
\( \varepsilon_k \) - volume fraction of phase \( k \)
\( \varepsilon_o \) - bulk porosity
\( \nu_k \) - kinematic viscosity of phase \( k \), cm\(^2\)/s
\( \xi \) - electroosmotic coefficient
\( \Pi_h \) - Peltier coefficient for charge-transfer reaction \( h \), V
\( \rho_k \) - density of phase \( k \), g/cm\(^3\)
\( \sigma_n \) - deviation of distribution \( n \)
\( \sigma_o \) - standard conductivity in the electronically conducting phase, S/cm
\( \eta_{h,k-p} \) - electrode overpotential of reaction \( h \) between phases \( k \) and \( p \), V
\( \eta^* \) - dimensionless overpotential
\( \eta_o(T) \) - dimensionless temperature-dependent growth parameter

\( \eta_{OHP} \) - local reaction overpotential, V

\( \theta \) - contact angle, degrees

\( \kappa \) - conductivity of the ionically conducting phase, S/cm

\( \lambda \) - moles of water per mole of sulfonic acid sites

\( \lambda_L \) - relative mobility of the liquid phase

\( \mu \) - viscosity, Pa-s

\( \mu_{ij}^{\text{eff}} \) - effective viscosity within a pore between nodes \( i \) and \( j \), kPa-s

\( \mu_i \) - (electro)chemical potential of species \( i \), J/mol

\( \mu_{nw} \) - non-wetting fluid viscosity, kPa-s

\( \mu_w \) - wetting fluid viscosity, kPa-s

\( \mu_i^\alpha \) - electrochemical potential of species \( i \) in phase \( \alpha \), J/mol

\( \tau \) - stress tensor, kPa

\( \tau_i \) - a limiting stochastic induction time, s

\( \tau_k \) - tortuosity of phase \( k \)

\( \phi \) - Thiele modulus, and gas-free volume fraction of ice within the pores,

\( \phi \) - phase shift of response in impedance

\( \Phi_k \) - electrostatic potential of phase \( k \), V

\( \psi \) - Dimensionless property

\( \psi_i \) - Permeation coefficient of species \( i \), mol/bar·cm·s

\( \Psi(\theta) \) - the contact angle distribution

\( \omega \) - energy parameter for oxide adsorption, J/mol, and frequency of sinusoidal signal, 1/s

**Subscripts/Superscripts**

1 - electronically conducting phase

2 - ionically conducting phase

agg - agglomerate
CL  - catalyst layer
COR  - carbon oxidation reaction
DON  - Donnan
eff  - effective value, corrected for tortuosity and porosity
ext  - external to the control volume
f    - fixed ionic site in the membrane
film - film covering the agglomerate
g    - homogeneous reaction number
G    - gas phase
h    - electron-transfer reaction number
H    - Stern or Helmholtz plane
HOR  - hydrogen-oxidation reaction
i    - generic species
ice  - ice portion in the pores
j    - generic species
k    - generic phase
l    - heterogeneous reaction number
L    - liquid phase
m    - mixture
OHP  - outer Helmholtz plane
ORR  - oxygen-reduction reaction
p    - generic phase
pore - cylindrical pore
PtO  - Pt oxide
ref  - parameter evaluated at the reference conditions
s    - solid phases
w    - water
∞    - bulk

Abbreviations
CFD  - computational fluid dynamics
DM   - diffusion media
EDL  - electric double layer
EIS   - electrochemical impedance spectroscopy
CGSG - Gouy-Chapman-Stern-Grahame
GDL  - gas-diffusion media
GISAXS - Grazing-incidence small-angle x-ray Scattering
HOR   - hydrogen oxidation reaction
JMAK  - Johnson-Mehl-Avrami-Kolmogorov
MEA   - membrane electrode assembly
MPL  - micro-porous layer
NSTF  - nano-structured thin film
OHP   - outer Helmholtz plane
ORR   - oxygen reduction reaction
PCI   - phase-change-induced
PEFC  - polymer electrolyte fuel cells
PEM   - polymer electrolyte membrane
PTLs  - porous transport layers
PZC   - potential of zero charge
UPD   - underpotential deposited
XCT   - computed x-ray tomography
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